TRANSIENT THERMOELASTIC PROBLEM FOR TWO REGION CONCENTRIC CYLINDER WITH PERFECT THERMAL CONTACT AT THE INTERFACE

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Abstract: This paper is a treatment of a transient thermoelastic contact problem in two region concentric solid cylinder with perfect thermal contact at the interface subjected to convectional boundary condition at the outer surface $r = b$. The solution meets the boundary condition on both the lateral surfaces.

In order to analyse the problem, dimensionless quantities are used to develop the analysis of temperature field. We have used the method of generalized integral transforms. The dimensionless displacements and stresses have been attained using Goodier's thermoelastic function ϕ and Love's function L .

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Key Words: concentric solid cylinders, perfect thermal contact at interface, dimensionless quantities, Goodier's thermoelastic function, Love's function

1. Introduction

Some interest has been shown in the solutions of contact problems involving the heating of solid surfaces such as J.R. Barber [3], [4], [5] considered indentation of semi-infinite solid in elastic half space. D.L. Elements and G.D. Toy [6] considered two contact problems in anisotropic thermoelasticity. Z.S. Olesiak [8] considered axially symmetric thermal contact stress for a micropolar elastic semi-space. O. Tamate et al. [10] considered problem of singularities at the ends

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of thin elastic stiffener bounded to a semi-infinite medium. R.D. Kulchytsky-Zhyhailo et al. [7] studied problem on thermal contact of two axially symmetric elastic solids. A.A. Yevtushenko and R.D. Kulchytsky-Zhyhailo [11] studied approximation solution of thermoelastic contact problem with frictional heating in general case. H.H. Sherif and K.A. Helmy [9] considered two dimensional problem for a half space in magnetothermoelasticity with thermal relaxation. G. Bonfanti et al. [1] studied existence exponential stability and dynamic contact problem of two thermoelastic beams. G. Bonfanti et al. [2] studied energy decay for a thermoelastic contact problem involving heat transfer.

This paper presents the thermoelastic contact problem of two region concentric cylinder with perfect thermal contact at interface. The purpose of this paper is to study heat transfer and thermoelastic effect in concentric regions. The results presented here have numerous applications in engineering of rocket liners, nuclear reactors etc.

To attain the solution of this problem, following attempts are made:

1. The solution for temperature field is obtained using orthogonal expansion technique over two layer region and Fourier Transforms.
2. Dimensionless quantities are used.
3. Goodier's thermelastic potential function ϕ and Love's function L is used to obtain solution for thermal displacements and stresses.
4. The solution of the problem is presented in terms of dimensionless temperature, dimensionless displacements and dimensionless stresses for two regions, graphically represented and conclusions are made accordingly.

2. Statement of the Heat Conduction Problem

Consider a two region solid cylinder of outer radius $r = b$ with an inner radius $r = a$ and there is a perfect thermal contact between two layers. The cylinders are initially at uniform temperature T_0 and for times $t \geq 0$ the boundary surface at $r = b$ dissipates heat by the convection into a surrounding at zero. There is no heat generation within the solid body under consideration.

The two-dimensional transient heat conduction equations are given as

$$\frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r} \frac{\partial T_1}{\partial r} + \frac{\partial^2 T_1}{\partial z^2} = \frac{1}{\alpha_1} \frac{\partial T_1}{\partial t} \quad \text{in } 0 \leq r \leq a, t > 0 \quad (1)$$

Figure 1: Shows the geometry of the problem

$$\frac{\partial^2 T_2}{\partial r^2} + \frac{1}{r} \frac{\partial T_2}{\partial r} + \frac{\partial^2 T_2}{\partial z^2} = \frac{1}{\alpha_2} \frac{\partial T_2}{\partial t} \quad \text{in } a \leq r \leq b, t > 0 \quad (2)$$

subject to boundary conditions

$$T_1 = T_2 = T a f(z) \quad \text{at interface } r = a, t > 0 \quad (3)$$

$$K_1 \frac{\partial T_1}{\partial r} = K_2 \frac{\partial T_2}{\partial r} \quad \text{at interface } r = a, t > 0 \quad (4)$$

$$K_2 \frac{\partial T_2}{\partial r} + h T_2 = 0 \quad \text{at the outer boundary condition } r = b, t > 0 \quad (5)$$

with initial conditions

$$T_1 = T_0 = 0 \quad \text{in } 0 \leq r \leq a, t = 0 \quad (6)$$

$$T_2 = T_0 = 0 \quad \text{in } a \leq r \leq b, t = 0, \quad (7)$$

where T_0 is the initial temperature, K_1 , K_2 be the thermal conductivities of the inner and outer region respectively, and α_1 and α_2 be the thermal diffusivities. Moreover, we use the following dimensionless quantities:

$$\widehat{T} = \frac{T}{T_a}, \quad t' = \frac{kt}{r_i^2}, \quad \rho = \frac{r}{r_i}, \quad \xi = \frac{z}{r_i}. \quad (8)$$

Now we consider axi-symmetrical thermoelastic problem in a two region solid circular cylinder by use of Love's function L and thermoelastic displacement potential ϕ . The displacement and stresses are given by

$$u_\rho = \frac{1}{r_i} \frac{\partial \phi_i}{\partial \rho} - \frac{1}{r_i^2} \frac{\partial L_i}{\partial \rho \partial \xi} \quad (9)$$

$$u_\xi = \frac{1}{r_i} \frac{\partial \phi_i}{\partial \xi} + 2(1 - \nu) \nabla^2 L_i - \frac{1}{r_i^2} \frac{\partial^2 L_i}{\partial \xi^2} \quad (10)$$

$$\frac{\bar{\rho}\bar{\rho}}{2G} = \frac{1}{r_i^2} \frac{\partial^2 \phi_i}{\partial \rho^2} - \nabla^2 \phi_i + \frac{1}{r_i} \frac{\partial \left(\nu \nabla^2 L_i - \frac{1}{r_i^2} \frac{\partial^2 L_i}{\partial \rho^2} \right)}{\partial \xi} \quad (11)$$

$$\frac{\bar{\theta}\bar{\theta}}{2G} = \frac{1}{\rho r_i^2} \frac{\partial \phi_i}{\partial \rho} - \nabla^2 \phi_i + \frac{1}{r_i} \frac{\partial \left(\nu \nabla^2 L_i - \frac{1}{\rho r_i^2} \frac{\partial L_i}{\partial \rho} \right)}{\partial \xi} \quad (12)$$

$$\frac{\bar{\xi}\bar{\xi}}{2G} = \frac{1}{r_i^2} \frac{\partial^2 \phi_i}{\partial \xi^2} - \nabla^2 \phi_i + \frac{1}{r_i} \frac{\partial \left((2 - \nu) \nabla^2 L_i - \frac{1}{r_i^2} \frac{\partial^2 L_i}{\partial \xi^2} \right)}{\partial \xi} \quad (13)$$

$$\frac{\bar{\rho}\bar{\xi}}{2G} = \frac{1}{r_i^2} \frac{\partial^2 \phi_i}{\partial \rho \partial \xi} + \frac{1}{r_i} \frac{\partial \left((1 - \nu) \nabla^2 L_i - \frac{1}{r_i^2} \frac{\partial^2 L_i}{\partial \xi^2} \right)}{\partial \xi}, \quad (14)$$

where $i = 1, 2$, G and ν are the shear modulus and Poisson's ratio respectively, and $\mu\rho$ and $\mu\xi$ are the displacements in the ρ and ξ directions, respectively.

The functions L and ϕ must satisfy the following differential equations:

$$\nabla^2 \phi_i = \frac{(1 + \nu)}{(1 - \nu)} \alpha T_i, \quad \nabla^4 L_i = 0. \quad (15)$$

3. Determination of Temperature Field

The temperature suitable to this problem with prescribed and initial and boundary conditions is given by

$$T_i(r, z, t) = A_0 + \int_0^\infty A(s) \left(I_0(sr) - \sum_{n=1}^\infty \frac{A_n X_{jn}(r)}{\alpha_j(s_m^2 + \beta_{jn}^2)} \right) \times (1 - e^{-a_j(s_m^2 + \beta_{jn}^2)t}) \cos sz \, ds, \quad t > 0, \quad (16)$$

where $j=1,2$, A_0 and $A(s)$ are unknown functions and $I_0(sr)$ denotes the modified Bessel function of order zero.

Using initial and boundary conditions we obtained the temperature function as

$$T_i = \int_0^\infty \frac{\bar{f}(s)}{I_0(sa)} \left(I_0(sr) - \sum_{n=1}^\infty \frac{A_n X_{jn}(r)}{\alpha_j(s_m^2 + \beta_{jn}^2)} \right)$$

$$\times (1 - e^{-a_j}(s_m^2 + \beta_{jn}^2)t) \cos sz \, ds, \quad t > 0, \quad (17)$$

where the eigenfunction $X_{jn}(r)$ for two layer problem under consideration is

$$X_{1n}(r) = J_0(\beta_{1n}r) \text{ in layer 1, i.e. } 0 \leq r \leq a, \quad (18)$$

$$X_{2n}(r) = B_n (J_0(\beta_{2n}r)) + C_n Y_0(\beta_{2n}r) \text{ in layer 2, i.e. } a \leq r \leq b. \quad (19)$$

The function $Y_0(\beta_{2n}r)$ is excluded from the solution for region 1 because it includes the origin.

The eigenvalues β_{1n} and β_{2n} are related by

$$\alpha_1 \beta_{1n}^2 = \alpha_2 \beta_{2n}^2.$$

Now unknown B_n , C_n and the eigenvalues β_{jn} can be obtained using boundary conditions (3), (4) and (5).

Using the boundary conditions (4), C_n can be obtained as

$$C_n = \frac{h_2 (J_0(\beta_{2n}b)) - K_2 \beta_{2n} J_1(\beta_{2n}b)}{K_2 \beta_{2n} (Y_1(\beta_{2n}b)) - h_2 Y_0(\beta_{2n}b)}.$$

Using the boundary condition (3) we obtain B_n as

$$B_n = \frac{J_0(\beta_{1n}a)}{(J_0(\beta_{2n}a)) + C_n Y_0(\beta_{2n}a)}.$$

Now A_n can be obtained using the relation

$$A_n = \frac{\sum_{i=1}^{\infty} W_i^2 \int_{\text{layer } i} F(x) X_{in}(x) W(x) dx}{\sum_{i=1}^{\infty} W_i^2 \int_{\text{layer } i} X_{in}^2(x) W(x) dx},$$

where

$$W_i^2 = \frac{k_1}{\alpha_1}, \quad W = r, \quad F_1 = T_0, \quad i = 1, 2,$$

$$X_{1n}(r) = J_0(\beta_{1n}r), \quad X_{2n}(r) = B_n [(J_0(\beta_{2n}r)) + C_n Y_0(\beta_{2n}r)],$$

we obtain A_n using the above equations as

$$A_n = \frac{\frac{K_1}{\alpha_1} \int_0^a T_0 J_0(\beta_{1n}r) r dr + \frac{K_2}{\alpha_2} \int_a^b T_0 B_n [J_0(\beta_{2n}r) + C_n Y_0(\beta_{2n}r)] r dr}{\frac{K_1}{\alpha_1} \int_0^a J_0^2(\beta_{1n}r) r dr + \frac{K_2}{\alpha_2} \int_a^b B_n^2 [J_0(\beta_{2n}r) + C_n Y_0(\beta_{2n}r)]^2 r dr},$$

$$A_n = \frac{2T_0}{\alpha \beta_{1n}} \left[J_1(\beta_{1n}a) + \frac{K_2}{K_1} \sqrt{\frac{\alpha_1}{\alpha_2}} \left[\frac{b}{a} \{ [B_n J_1(\beta_{2n}b) + C_n Y_1(\beta_{2n}b)] r \} \right] \right]$$

$$\begin{aligned}
& - [B_n J_1(\beta_{2n}a) + C_n Y_1(\beta_{2n}a)] \} \Big] \left[J_0^2(\beta_{1n}a) + J_1^2(\beta_{1n}a) \right. \\
& \quad + \frac{K_2 \alpha_1}{K_1 \alpha_2} \left[\frac{b^2}{a^2} \times \left\{ [B_n (J_0(\beta_{2n}b) + C_n Y_0(\beta_{2n}b))]^2 \right. \right. \\
& \quad + [B_n (J_1(\beta_{2n}b) + C_n Y_1(\beta_{2n}b))]^2 \Big\} - \left\{ [B_n (J_0(\gamma_n a) + C_n Y_0(\gamma_n a))]^2 \right. \\
& \quad \left. \left. + [B_n (J_1(\beta_{2n}a) + C_n Y_1(\beta_{2n}a))]^2 \right\} \right] \Big]^{-1},
\end{aligned}$$

and the finite Fourier Transform $T_a f(z)$ is defined as

$$\bar{f}(s) = \int_0^l T_a f(z) \cos sz \, dz.$$

Now the temperatures for different layers (i.e. $0 \leq r \leq a$ as layer 1 and $a \leq r \leq b$ as layer 2) are:

$$\begin{aligned}
T_1(r, z, t) = \sum_{m=1}^{\infty} \frac{T_a \bar{f}(s_m)}{I_0(s_m a)} \left[I_0(s_m r) - \sum_{n=1}^{\infty} \frac{A_n J_0(\beta_{1n} r)}{\alpha_1 (s_m^2 + \beta_{1n}^2)} \right] \\
\times (1 - e^{-\alpha_1 (s_m^2 + \beta_{1n}^2) t}) \cos s_m z, \quad (20)
\end{aligned}$$

$$\begin{aligned}
T_2(r, z, t) = \sum_{m=1}^{\infty} \frac{T_a \bar{f}(s_m)}{I_0(s_m a)} \left[I_0(s_m r) - \sum_{n=1}^{\infty} \frac{A_n [B_n (J_0(\beta_{2n} r) + C_n Y_0(\beta_{2n} r))] }{\alpha_2 (s_m^2 + \beta_{2n}^2)} \right] \\
\times (1 - e^{-\alpha_2 (s_m^2 + \beta_{2n}^2) t}) \cos s_m z. \quad (21)
\end{aligned}$$

Using the dimensionless quantities from (8) in equations (20) and (21), we obtain dimensionless temperature for two layers (i.e. $0 \leq \rho \leq a$ as layer 1, and $a \leq \rho \leq b$ as layer 2) as

$$\begin{aligned}
T_1(\rho, \xi, t') = \sum_{m=1}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I_0(s_m \rho a) - \sum_{n=1}^{\infty} \frac{A_n J_0(\beta_{1n} \rho a)}{\alpha_1 (s_m^2 + \beta_{1n}^2)} \right] \\
\times \left(1 - e^{-\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}} \right) \cos s_m \xi a, \quad (22)
\end{aligned}$$

$$\begin{aligned}
T_2(\rho, \xi, t') = \sum_{m=1}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I_0(s_m \rho b) - \sum_{n=1}^{\infty} \frac{A_n [B_n (J_0(\beta_{2n} \rho b) + C_n Y_0(\beta_{2n} \rho b))] }{\alpha_2 (s_m^2 + \beta_{2n}^2)} \right] \\
\times \left(1 - e^{-\alpha_2 (s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}} \right) \cos s_m \xi b. \quad (23)
\end{aligned}$$

4. Determination of Displacement Potential and Love's function

Using equations (22) and (23) in equation (15), the displacement potential and Love's function is given as

$$\begin{aligned}\nabla^2\phi &= \frac{1+\nu}{1-\nu}\alpha T, \\ \phi_1 &= \frac{1+\nu}{1-\nu}\alpha_1^2\frac{a^2}{K}T_a\sum_{m=1}^{\infty}\frac{\bar{f}(s_m)}{I_0(s_ma)}\left[I_0(s_m\rho a)-\sum_{n=1}^{\infty}\frac{A_nJ_0(\beta_{1n}\rho a)}{\alpha_1(s_m^2+\beta_{1n}^2)}\right] \\ &\quad \times\left(t'+\frac{e^{-\alpha_1(s_m^2+\beta_{1n}^2)}\frac{a^2t^1}{K}}{\alpha_1(s_m^2+\beta_{1n}^2)\frac{a^2}{K}}\right)\cos s_m\xi a.\end{aligned}\quad (24)$$

Similarly,

$$\begin{aligned}\phi_2 &= \frac{1+\nu}{1-\nu}\alpha_2^2\frac{b^2}{K}T_a\sum_{m=0}^{\infty}\frac{\bar{f}(s)}{I_0(s_ma)} \\ &\quad \times\left[I_0(s_m\rho b)-\sum_{n=1}^{\infty}\frac{A_n}{\alpha_2(s_m^2+\beta_{2n}^2)}[B_n[J_0(\beta_{2n}\rho b)+C_nY_0(\beta_{2n}\rho b)]]\right] \\ &\quad \times\left(t'+\frac{e^{-\alpha_2(s_m^2+\beta_{2n}^2)}\frac{b^2t^1}{K}}{\alpha_2(s_m^2+\beta_{2n}^2)\frac{b^2}{K}}\right)\cos s_m\xi b.\end{aligned}\quad (25)$$

Now Love's function is given as

$$\begin{aligned}L_1 &= \frac{1+\nu}{1-\nu}\alpha_1^2\frac{a^2}{K}T_a \\ &\quad \times\sum_{m=1}^{\infty}\frac{\bar{f}(s_m)}{s_m^2}[D_nI_0(s_m\rho a)+E_ns_m\rho aI_1(s_m\rho a)]\sin s_m\xi a\end{aligned}\quad (26)$$

and

$$\begin{aligned}L_2 &= \frac{1+\nu}{1-\nu}\alpha_2^2\frac{b^2}{K}T_a \\ &\quad \times\sum_{m=1}^{\infty}\frac{\bar{f}(s_m)}{s_m^2}[D_nI_0(s_m\rho b)+E_ns_m\rho bI_1(s_m\rho b)]\sin s_m\xi b.\end{aligned}\quad (27)$$

5. Determination of Thermal Displacements and Stresses for Region 1

Using equations (24) and (26) in equations (9) to (10), the thermal displacements U_ρ are given as

$$\begin{aligned}
 U_\rho = & \frac{1+\nu}{1-\nu} \alpha_1 \frac{a^2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \\
 & \times \left[I_0'(s_m \rho a) s_m - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} J_0'(\beta_{1n} \rho a) \beta_{1n} a \right] \\
 & \times \left(t' + \frac{e^{-\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) \cos s_m \xi a \\
 & - \frac{1+\nu}{1-\nu} \frac{\alpha_1}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} \\
 & \times [D_n I_0'(s_m \rho a) s_m a + E_n s_m \rho I_1'(s_m \rho a) s_m a + I_1(s_m \rho a) s_m a] \\
 & \times \cos s_m \xi a.
 \end{aligned} \tag{28}$$

Similarly, we can get thermal displacement U_ξ .

Using equations (24) and (26) in equations (11) to (14), the thermal stresses are given as

$$\begin{aligned}
 \frac{\bar{\rho}\bar{\rho}}{2G} = & \frac{1+\nu}{1-\nu} \alpha_1 \frac{a^2}{K} T_a \\
 & \times \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0''(s_m \rho a) s_m^2 \\
 & - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} J_0''(\beta_{1n} \rho a) \beta_{1n}^2] \\
 & \times \left(t' + \frac{e^{-\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) \cos s_m \xi a \\
 & - \left\{ \frac{1+\nu}{1-\nu} \alpha_1 \frac{a^2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0''(s_m \rho a) s_m^2] \right. \\
 & \left. - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} J_0''(\beta_{1n} \rho a) \beta_{1n}^2 \right]
 \end{aligned}$$

$$\begin{aligned}
& \times \left(t' + \frac{e^{-\alpha_1(s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) \cos s_m \xi a \\
& + \frac{1}{\rho} \frac{1 + \nu}{1 - \nu} \alpha_1 \frac{a}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0'(s_m \rho a) s_m^2 \\
& - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} J_0'(\beta_{1n} \rho a) \beta_{1n}^2] \\
& \times \left(t' + \frac{e^{-\alpha_1(s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) \cos s_m \xi a \\
& - \frac{1 + \nu}{1 - \nu} \alpha_1 \frac{a^2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0(s_m \rho a) s_m^2 \\
& - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} J_0(\beta_{1n} \rho a)] \\
& \times \left(t' + \frac{e^{-\alpha_1(s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) (\cos s_m \xi a) s_m^2 \\
& + \left\{ \frac{\nu(1 + \nu)}{(1 - \nu)} \alpha_1 \frac{a^2}{K} \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I_0''(s_m \rho a) \right. \\
& + \rho I_1''(s_m \rho a) s_m a + 2I_1'(s_m \rho a)] \cos s_m \xi a s_m \\
& + \frac{1}{\rho} \frac{\nu(1 + \nu)}{(1 - \nu)} \alpha_1 \frac{a}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0'(s_m \rho a) s_m \\
& + E_n s_m \rho I_1'(s_m \rho a) s_m a + I_1(s_m \rho a) \\
& - \frac{\nu(1 + \nu)}{(1 - \nu)} \alpha_1 \frac{a^2}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I_0(s_m \rho a) \\
& + E_n s_m \rho a I_1(s_m \rho a)] \cos s_m \xi a s_m \} \\
& + \frac{(1 + \nu)}{(1 - \nu)} \alpha_1 \frac{a^2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0''(s_m \rho a) s_m^2 \\
& + E_n s_m \rho I_1''(s_m \rho a) s_m^2 + I_1'(s_m \rho a) s_m + I_1'(s_m \rho a) s_m] \cos s_m \xi a s_m, \quad (29)
\end{aligned}$$

$$\frac{\bar{\rho} \bar{\xi}}{2G} = \frac{(1 + \nu)}{(1 - \nu)} \frac{\alpha_1}{K} T_a$$

$$\begin{aligned}
& \times \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I'_0(s_m \rho a) s_m a - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_1 (s_m^2 + \beta_{1n}^2)} a^2 J'_0(\beta_{1n} \rho a) \right] \\
& \times \left(t' + \frac{e^{-\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2 t'}{K}}}{\alpha_1 (s_m^2 + \beta_{1n}^2) \frac{a^2}{K}} \right) \sin(s_m \xi a) s_m a \\
& + \frac{(1+\nu)}{(1-\nu)} \alpha_1 \frac{a}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I''_0(s_m \rho a) s_m a \\
& + E_n \{ I'''_1(s_m \rho a) s_m a^2 + I''_1(s_m \rho a) s_m a + 2I'_1(s_m \rho a) a \}] \sin(s_m \xi a) \\
& + \frac{1}{\rho} \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_1}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I''_0(s_m \rho a) a^2 \\
& + E_n \{ I''_1(s_m \rho a) s_m a^2 + I'_1(s_m \rho a) a + I'_1(s_m \rho a) a \}] \sin(s_m \xi a) \\
& - \frac{1}{\rho^2} \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_1}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m} [D_n I'_0(s_m \rho a) a \\
& + E_n \{ \rho I'_1(s_m \rho a) s_m a + I_1(s_m \rho a) \}] \sin(s_m \xi a) \\
& - \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_1}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I'_0(s_m \rho a) s_m a \\
& + E_n s_m \{ \rho a I'_1(s_m \rho a) s_m a + a I_1(s_m \rho a) \}] \sin(s_m \xi a) \\
& + \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_1}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I'_0(s_m \rho a) s_m a \\
& + E_n s_m a \{ \rho I'_1(s_m \rho a) s_m a + I_1(s_m \rho a) \}] \sin(s_m \xi a). \tag{30}
\end{aligned}$$

Similarly we can obtain the stresses $\sigma_{\xi\xi}$ and $\sigma_{\theta\theta}$.

6. Determination of Thermal Displacements and Stresses for Region 2

Using equations (25) and (27) in equations (9) and (10), the thermal displacements for region 2 are given as

$$\begin{aligned}
U_\rho = & \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_2}{K} T_a \left\{ \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I'_0(s_m \rho b) s_m] \right. \\
& \left. - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2 (s_m^2 + \beta_{2n}^2)} [B_n \{ J'_0(\beta_{2n} \rho b) \beta_{2n} + C_n Y'_0(\beta_{2n} \rho b) \} \beta_{2n}] \right\}
\end{aligned}$$

$$\begin{aligned}
& \times \left(t' + \frac{e^{-\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \cos(s_m \xi b) \\
& - \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m} [D_n I_0'(s_m \rho b) s_m b \\
& + E_n s_m \{ \rho I_1'(s_m \rho b) s_m b + I_1(s_m \rho b) \}] \cos s_m \xi b \}.
\end{aligned} \tag{31}$$

Similarly we can get displacement U_ξ .

Using (27) and (30) in equations (11) to (14), the thermal stresses are given as

$$\begin{aligned}
\frac{\bar{\rho}\bar{\rho}}{2G} = & \frac{(1+\nu)}{(1-\nu)} b^2 \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I_0''(s_m \rho b) s_m^2 \right. \\
& \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2(s_m^2 + \beta_{2n}^2)} [B_n \{ J_0''(\beta_{2n} \rho b) \beta_{2n}^2 + C_n Y_0''(\beta_{2n} \rho b) \} \beta_{2n}^2] \Big] \\
& \times \left(t' + \frac{e^{-\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \cos s_m \xi b \\
& - \left\{ \frac{(1+\nu)}{(1-\nu)} b^2 \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0''(s_m \rho b) s_m^2 \right. \\
& \left. - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2(s_m^2 + \beta_{2n}^2)} [B_n \{ J_0''(\beta_{2n} \rho b) \beta_{2n}^2 + C_n Y_0''(\beta_{2n} \rho b) \} \beta_{2n}^2] \right] \\
& \times \left(t' + \frac{e^{-\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \cos s_m \xi b \\
& + \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_2}{\rho K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I_0'(s_m \rho b) s_m b \right. \\
& \left. - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2(s_m^2 + \beta_{2n}^2)} [B_n \{ J_0'(\beta_{2n} \rho b) \beta_{2n} b + C_n Y_0'(\beta_{2n} \rho b) \} \beta_{2n} b] \right] \\
& \times \left(t' + \frac{e^{-\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2(s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \cos s_m \xi b
\end{aligned}$$

$$\begin{aligned}
& -\frac{(1+\nu)}{(1-\nu)} \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} \left[I_0(s_m \rho b) \right. \\
& \left. - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2 (s_m^2 + \beta_{2n}^2)} [B_n \{J_0(\beta_{2n} \rho b) + C_n Y_0(\beta_{2n} \rho b)\}] \right] \\
& \times \left(t' + \frac{e^{-\alpha_2 (s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2 (s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \cos(s_m \xi b) s_m \\
& + \left\{ \frac{\nu(1+\nu)}{(1-\nu)} \frac{\alpha_2 b}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0''(s_m \rho b) s_m^2 \right. \\
& + E_n s_m^2 [\rho I_1''(s_m \rho b) + 2I_1'(s_m \rho b)]] \cos(s_m \xi b) s_m b \\
& + \frac{1}{\rho} \frac{\nu(1+\nu)}{(1-\nu)} \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0'(s_m \rho b) s_m \\
& + E_n s_m [\rho I_1'(s_m \rho b) s_m b + I_1(s_m \rho b)]] \cos(s_m \xi b) s_m b \\
& - \frac{\nu(1+\nu)}{(1-\nu)} \frac{\alpha_2 b}{K} T_a \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I_0(s_m \rho b) \\
& + E_n s_m [\rho b I_1(s_m \rho b)]] \cos(s_m \xi b) s_m b \\
& - \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0''(s_m \rho b) s_m^2 b^2 \\
& + E_n s_m [\rho I_1''(s_m \rho b) s_m^2 b^2 + 2s_m b I_1'(s_m \rho b)]] \cos(s_m \xi b) s_m \\
& - \frac{(1+\nu)}{(1-\nu)} \frac{\alpha_2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0''(s_m \rho b) s_m^2 b^2 \\
& + E_n s_m [\rho I_1''(s_m \rho b) s_m^2 b^2 + 2s_m b I_1'(s_m \rho b)]] \cos(s_m \xi b) s_m, \tag{32}
\end{aligned}$$

$$\begin{aligned}
\frac{\bar{\rho} \bar{\xi}}{2G} &= \frac{-(1+\nu)}{(1-\nu)} \frac{\alpha_2 b^2}{K} T_a \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{I_0(s_m a)} [I_0'(s_m \rho b) s_m \\
& - \sum_{n=1}^{\infty} \frac{A_n}{\alpha_2 (s_m^2 + \beta_{2n}^2)} [B_n \{J_0'(\beta_{2n} \rho b) + C_n Y_0'(\beta_{2n} \rho b) \beta_{2n}\}]] \\
& \times \left(t' + \frac{e^{-\alpha_2 (s_m^2 + \beta_{2n}^2) \frac{b^2 t'}{K}}}{\alpha_2 (s_m^2 + \beta_{2n}^2) \frac{b^2}{K}} \right) \sin(s_m \xi b) s_m
\end{aligned}$$

$$\begin{aligned}
& + (1 + \nu) T_a \frac{\alpha_2}{K} \left\{ b \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0'''(s_m \rho b) s_m^3 b + \right. \\
& + E_n s_m^2 [\rho I_1'''(s_m \rho b) s_m^3 b^2 + I_1''(s_m \rho b) s_m b] \\
& + 2 I_1''(s_m \rho b) s_m b] \sin s_m \xi b \\
& + \frac{1}{\rho} \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0''(s_m \rho b) s_m^2 b \\
& + E_n s_m [\rho I_1''(s_m \rho b) s_m^2 b^2 + \{I_1'(s_m \rho b) s_m b\}]] \sin s_m \xi b \\
& - \frac{1}{\rho^2} \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0'(s_m \rho b) s_m \\
& + E_n s_m [\rho I_1'(s_m \rho b) s_m b + I_1(s_m \rho b)]] \sin s_m \xi b \\
& - b \sum_{m=0}^{\infty} \bar{f}(s_m) [D_n I_0'(s_m \rho b) s_m b] \\
& + E_n s_m b [\rho I_1'(s_m \rho b) s_m b + I_1(s_m \rho b)] \sin s_m \xi b \} \\
& + \frac{(1 + \nu)}{(1 - \nu)} \frac{\alpha_2 b}{K} T_a \left\{ \sum_{m=0}^{\infty} \frac{\bar{f}(s_m)}{s_m^2} [D_n I_0'(s_m \rho b) s_m b \right. \\
& + E_n s_m [\rho I_1'(s_m \rho b) s_m b + I_1(s_m \rho b)]] \sin(s_m \xi b) s_m^2. \quad (33)
\end{aligned}$$

Similarly we can obtain the stresses $\sigma_{\xi\xi}$ and $\sigma_{\theta\theta}$.

7. Numerical Results and Discussion

Setting the following values for pure copper in both layers of equations (28) through (33), and solving them numerically and graphically, the patterns in the graphs have been analyzed and conclusions have been drawn accordingly:

$$h = 0.1, \quad \alpha = 16.5 \times 10^{-6} (J/K) = \alpha_1 = \alpha_2,$$

$$\nu = 0.35, \quad \lambda = 386 W/m/k, G = 44.7 GPa,$$

$$K_1 = K_2 = 117 \times 10^6 (m^2/s), a = 1.5, b = 1.8,$$

$$\xi = -0.1 \cdots +1.1, \quad A_n = 0.5, B_n = 1.1, C_n = 1.5, D_n = 1.2, E_n = 1.3.$$

s_m are roots of $\cos(sz) = 0$, β_{1n} are roots of $J_0(\beta_{1n}a) = 0$,

β_{2n} are roots of

$$h_2(J_0(\beta_{2n}b)) - K_2 \beta_{2n} J_1(\beta_{2n}b) = 0,$$

$$f(\xi) = H(\xi_0 - |\xi|)$$

for $\xi_0 = 1$ we take $\bar{f}(s_m) = 0.1 \dots 0.9$.

To examine the influence of heating on the behavior of thermoelastic problem of two region concentric cylinders with copper as its material, we have performed numerical calculations. For convenience we have used dimensionless quantities and numerical results are presented here for $-0.1 \leq \xi \leq 1.1$, $\rho = 1.5$ for region 1 and $\rho = 1.8$ for region 2. In this case the function $f(\xi)$ is expressed as

$$f(\xi) = H(\xi_0 - |\xi|)$$

for $\xi_0 = 1$.

According to numerical results, graphs are plotted for dimensionless temperature distribution, dimensionless stresses for both the regions. Figures 2 and 3 show temperature distribution for region 1 and region 2 in radial direction. In both the regions initially temperature increases in the vicinity of $\xi = 0$, then gradually decreases with increasing values of ξ while it shows variation with increasing values of ρ . This effect is observed due to factor $f(\xi)$ in $T_a f(\xi)$ expressed as Heaviside step function $f(\xi) = H(\xi_0 - |\xi|)$ and the thermal contact at interface. Figures 4 and 5 show radial displacement for region 1 and region 2 in axial and radial direction. In region 1, radial displacement in ξ direction shows initial tension and then turns to compression as ξ increases, while in region 2 it shows a sinusoidal pattern. This is due to convectional boundary condition at outer boundary. Radial displacement in both regions in radial direction increases with the increasing values of ρ due to heat flux. Figures 6 and 7 show radial stress $\rho\rho/2G$ for region 1 and region 2 in axial directions. The figures show that the value of radial stresses in both the regions turns from negative to positive values in the vicinity of $\xi=0$ and gradually increases in increasing direction of ξ and ρ . This is due to continuity of heat flux at the perfect thermal contact of two regions. Figures 8 and 9 show axial stresses for both the regions in axial ξ direction. In region 1, axial stresses $\xi\xi/2G$ initially increases with high intensity and then gradually decreases while in region 2, axial stresses $\xi\xi/2G$ decrease to negative values. This occurs due to heat convection at the boundary of region 2.

Figure 2: $T_1(\rho, \xi, t')$ versus ρ for different values of ξ in region 1

Figure 3: $T_2(\rho, \xi, t')$ versus ρ for different values of ξ in region 2

Figure 4: U_ρ versus ξ for different values of ρ in region 1

Figure 5: U_ρ versus ρ for different values of ξ in region 2

Figure 6: $\rho\rho/2G$ versus ξ for different values of ρ in region 1

Figure 7: $\rho\rho/2G$ versus ξ for different values of ρ in region 2

Figure 8: $\xi\xi/2G$ versus ξ for different values of ρ in region 1

Figure 9: $\xi\xi/2G$ versus ξ for different values of ρ in region 2

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