THE PHYSICAL MOTION OF INTRINSIC SPIN AND THE EXPERIMENTAL IMPLICATION

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Abstract: We propose a revised Stern-Gerlach experiment of setting up two perpendicular magnetic fields $B(1)$ and $B(2)$; we predict that half of the electrons will be observed to align with $B(1)$ and the other half, $B(2)$, so that any electron, left (right) - handed, has four spin states, up (down), left (right), down (up), and right (left), in a 720° -cycle, altering the basis for quantum computing.

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1. Introduction and Motivation

Quantum computation has been resting on the basic tenet of the existence of two spin states, up or down, for the electron (cf. e.g., [4], [5]). This paper re-examines this assertion. From Einstein's mass-shell equation

$$E^2 = (pc)^2 + (m_0c^2)^2.$$

Dirac derived the equation

$$\left(i\hbar \frac{\partial}{\partial t} - i\hbar c \sigma_i \nabla \right) (\psi_{up}, \psi_{down}) = m_0 c^2 (\psi_{up}, \psi_{down})$$

for a free electron, which rendered in Feynman ([3], p. 10-13, with notation adapted to this paper) the following set of differential equations

$$i\hbar \frac{d}{dt} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} = -\mu \|\mathbf{B}\| \sigma_z \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} \quad (1)$$

for an electron situated in a magnetic field $\mathbf{B} = (0, 0, B_z > 0)$, with the electron rest mass suppressed to zero. Since the two columns of the Pauli matrix

$$\sigma_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

represent two orthogonal vectors that do not correspond to a pair of up and down directions as in a magnetic dipole moment, we thus surmise that the actual wave motion of the electron in the entire process is:

$$up, left, down, \text{ and } right,$$

in a 720° – *cycle*, with (*up, left*) accounting for $\frac{\hbar}{2}$ in the first 360° turn and (*down, right*) accounting for $\frac{\hbar}{2}$ in the second 360° turn; i.e., there exist *four* spin states in a 720° – *cycle* for all *spin* – $\frac{1}{2}$ particles. In this way, (*up* $\perp$ *left*) or (*down* $\perp$ *right*) would correspond to the two orthogonal columns of σ_z . As such, we propose a revised Stern-Gerlach experiment that sets up two perpendicular magnetic fields to test our hypothesis.

An extensive literature research has yielded few results on the subject of intrinsic spin possibly because this construct has become a standard pillar in quantum physics (cf. [2] for an earlier study of this concept, [1] for a study of the spin structure of the nucleon, and [6] for observed 3 – *dimensional* spinning motions). Section 2 below will present our hypothesis in details and Section 3 will conclude with a summary remark.

2. A Hypothetical Motion of Electron Spins in Euclidean $\mathbb{R}^3$

We begin with the following short-hand notation for easy visualization:

$$\begin{aligned} E &\equiv East \equiv (1, 0, 0), \quad W \equiv West \equiv (-1, 0, 0), \\ N &\equiv North \equiv (0, 1, 0), \quad S \equiv South \equiv (0, -1, 0), \\ T &\equiv Top \equiv (0, 0, 1), \quad B \equiv Bottom \equiv (0, 0, -1), \end{aligned}$$

relative to any frame in question. We hypothesize that any left-handed electron e_L has the following spinning motion relative to a laboratory frame (the right-handed electron e_R would have the time-reversed spinning motions):

$$S \rightarrow E \rightarrow N \rightarrow B \rightarrow S$$

in a 360° – *cycle* composed of: (1) a 180° – *rotation* along the (x, y) – *plane*, and (2) a 180° – *rotation* along the (y, z) – *plane* with e_L ended upside down.

The angular momenta of e_L are thus:

(1) $\frac{\hbar}{2}$ by the motion of $S \rightarrow E \rightarrow N$, a counterclockwise rotation relative to the z – *axis* of e_L , which is assigned a special label T_{e_L} (connoting the Top of e_L as bounded to the moving frame of e_L), and

(2) $\frac{\hbar}{2}$ by the motion of $N \rightarrow B \rightarrow S$, a counterclockwise rotation relative to the x – *axis* of e_L , which is assigned a special label E_{e_L} (connoting the East of e_L as bounded to the moving frame of e_L), which however has been rotated by the above motion (1) from E to W , so that in the laboratory frame motion (2) becomes clockwise while motion (1) remains to be counterclockwise.

Consider an externally imposed magnetic field

$$\begin{aligned} \mathbf{B} &= \|\mathbf{B}\| (-\cos \theta_B, 0, \sin \theta_B) \text{ (tesla)}, \\ 0 &\leq \theta_B \leq \frac{\pi}{2}. \end{aligned}$$

Then motion (1) has wavefunction, with $\omega, t > 0$,

$$C_1(t) \equiv \psi_2(t) = \sin \theta_B e^{i\omega t},$$

and motion (2) has wavefunction

$$C_2(t) \equiv \psi_1(t) = \cos \theta_B e^{-i\omega t}.$$

The above $(C_1(t), C_2(t))$ thus serve as the basis directions for the Pauli matrix

$$\begin{aligned} \sigma_z &\equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ with} \\ i\hbar \frac{d}{dt} \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix} &= -\frac{\mu}{2} \|\mathbf{B}\| \sigma_z \begin{pmatrix} \psi_1(t) \\ \psi_2(t) \end{pmatrix}; \end{aligned} \quad (2)$$

here in Equation (2), $\mu \equiv$ electron magnetic dipole moment $\approx -9.28 \times 10^{-24}$ *joule/tesla* is *divided by 2* in contrast to the whole factor μ as contained in the original Equation (1) by Feynman for we are hypothesizing that Equation (1)

is based on a 720° – *cycle* and Equation (2) is based on a 360° – *cycle*; also classically

$$\begin{aligned}\mu \times \mathbf{B} &= \tau \equiv \text{torque}, \text{ and} \\ |\tau| &= |\mu| \cdot |\mathbf{B}| \cdot \sin \theta\end{aligned}$$

is the energy needed to displace μ from $\mathbf{B}$ by θ . At $\theta_B = 0$, the laboratory is to observe E_{e_L} at W ; at $\theta_B = \frac{\pi}{2}$, the laboratory is to observe T_{e_L} at T . In general,

$$\begin{aligned}\text{Prob}(E_{e_L} \text{ at } W) &= \cos^2 \theta_B \text{ and} \\ \text{Prob}(T_{e_L} \text{ at } T) &= \sin^2 \theta_B;\end{aligned}$$

in either event the observed angular momentum is $\hbar/2$ due to $180^\circ = 360^\circ/2$.

We thus propose a Stern-Gerlach experiment of setting up two magnetic fields,

$$\begin{aligned}\mathbf{B}_1 &= \|\mathbf{B}\| (0, 0, 1) \text{ and} \\ \mathbf{B}_2 &= \|\mathbf{B}\| (-1, 0, 0);\end{aligned}$$

we predict that half of the electrons will be observed to align with $\mathbf{B}_1$ and the other half, $\mathbf{B}_2$.

After the above first 360° rotation, the second 360° rotation is to have E_{e_L} observed at E or T_{e_L} observed at B . Setting $\theta_B = \frac{\pi}{2}$, one then has, over the entire 720° – *cycle*,

$$\begin{aligned}\text{Prob}(T_{e_L} \text{ at } T) &\lesssim \frac{1}{2} \lesssim \text{Prob}(T_{e_L} \text{ at } B), \text{ or} \\ \text{Prob}(Up) &\lesssim \frac{1}{2} \lesssim \text{Prob}(Down),\end{aligned}$$

where the “ $\lesssim$ ” sign reflects the fine-structure, with spin-up carrying an angular momentum $\hbar - \delta$ and spin-down, $\hbar + \delta$, so that they sum to $2\hbar$ over the 720° – *cycle*. One then would recover Equation (1),

$$\begin{aligned}&i\hbar \frac{d}{dt} \begin{pmatrix} \psi_2(t), \omega t \in [2\pi, 4\pi] \\ \psi_2(t), \omega t \in [0, 2\pi] \end{pmatrix} \\ &= -\mu \|\mathbf{B}\| \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_2(t), \text{cycle } 2 \\ \psi_2(t), \text{cycle } 1 \end{pmatrix},\end{aligned}$$

where the first column of σ_z , being orthogonal to the second column, would have to serve as an abstract label (as in heads/tails in Bernoulli trials of a coin

tossing) for the physical observation of T_{e_L} at $B \equiv (0, 0, -1)$. This 180° separation between $T \equiv (0, 0, 1)$ and $B \equiv (0, 0, -1)$ for the up-down interpretation of the quantum two states is necessary for a correspondence with magnetic *dipole* moment, but one finds vagueness in the literature on the definition of “up” and “down”, whether referring to abstract probability outcomes or a physical separation of 180° , and our above analysis clarifies this ambiguity.

3. Summary Note

The significance of our above hypothesis is that if it is experimentally verified, then any electron, e_L/e_R , has *four* spin states, up/down, left/right, down/up, and right/left (instead of the established two states, up and down), in a 720° – *cycle*, altering the foundation of quantum mechanics as well as computing. We thus suggest physics laboratories re-conduct Stern-Gerlach experiment as specified herein at the earliest possible times.

References

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