COMPLETE INTERSECTION CALABI-YAU NINE-FOLDS

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Abstract: In this paper a complete intersection Calabi-Yau 9-folds are considered. Their Hodge diamond, Todd classes and Chern characters for sheaves of differential k -forms are calculated.

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1. Introduction

All definitions in this article are conventional, as in [1]. We work in projective space $\mathbb{P}^n$ over an algebraically closed field of arbitrary characteristic. Let x be divisor degree of hypersurface $X \in \mathbb{P}^n$. We denote $X_m = \bigcap_{i=1}^k S_i^{s_i} \in \mathbb{P}^n$ as m -fold, which is a complete intersection of k hypersurfaces S_i degree s_i . Then X is a Calabi-Yau if $x = \sum s_i = n + 1$. For a sheaf of differential forms $\Omega_X^i = \Lambda^i \Omega_X$ we introduce Hodge numbers $h^{ij} = \dim H^i(\Omega_X^j)$, which not only are symmetrical: $h^{ij} = h^{ji}$, but Serre symmetrical also: $h^{ij} = h^{n-i, n-j}$. If we have Hodge numbers then Betti numbers may be calculated as

$$b_k = \sum_{i+j=k} h^{ij}.$$

We can also define the Euler characteristic of X as the alternating sum of the Betti numbers:

$$\chi(X) = \sum_k (-1)^k b_k.$$

For clarity, we rotate the matrix h^{ij} on 45° and call it as the Hodge diamond. So, for $n = 3$, we will have:

$$\begin{array}{ccccccc}
 & & & & h^{00} & & b_0 = h^{00} \\
 & & & & h^{10} & & b_1 = h^{10} + h^{01} \\
 & & & h^{11} & h^{01} & & b_2 = h^{20} + h^{11} + h^{02} \\
 & h^{20} & & h^{11} & h^{02} & & b_3 = h^{30} + h^{21} + h^{12} + h^{03} \\
 h^{30} & & h^{21} & & h^{12} & h^{03} & b_4 = h^{31} + h^{22} + h^{13} \\
 & h^{31} & & h^{22} & h^{13} & & b_5 = h^{32} + h^{23} \\
 & & h^{32} & & h^{23} & & b_6 = h^{33} \\
 & & & h^{33} & & &
 \end{array}$$

If $X = \bigcap_{k=1}^t S_k$ is complete intersection of t -hypersurfaces of degree s_k , then $N_X = \bigoplus_{k=1}^t \mathcal{O}_X(-s_k)$ and the sheaf $\mathcal{O}_X$ can be determined with used the following recurrence relations:

$$\begin{aligned}
 0 - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_{k-1}}(-s_k) - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_{k-1}} - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_k} &= 0 \\
 0 - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_{k-2}}(-s_{k-1}) - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_{k-2}} - \mathcal{O}_{S_1 \cap S_2 \cap \dots \cap S_{k-1}} &= 0 \\
 &\dots \\
 0 - \mathcal{O}_{S_1}(-s_2) - \mathcal{O}_{S_1} - \mathcal{O}_{S_1 \cap S_2} &= 0 \\
 0 - \mathcal{O}_{\mathbb{P}^n}(-s_1) - \mathcal{O}_{\mathbb{P}^n} - \mathcal{O}_{S_1} &= 0
 \end{aligned}$$

Now, for calculations of $H^i(\Omega_X^k)$ we can get the power series

$$0 - \text{Sym}^k N_X - \text{Sym}^{k-1} N_X \otimes \Omega_{\mathbb{P}^n|X} - \dots - \Omega_{\mathbb{P}^n|X}^k - \Omega_X^k = 0.$$

2. Todd and Chern classes

As shown in [2] for any rational complete intersection Calabi-Yau X all non-zero entries of the Hodge diamond are always lying on its equator or on the central column. Also $h^{ii} = 1$ if $i \neq m/2$. Therefore, we can simplify all calculations with use of characteristic classes theory.

We take the Riemann-Roch-Hirzebruch equation

$$\chi(E, X) = \int_X \text{ch}(E) \wedge \text{td}(T_X), \quad (1)$$

attach it to $E = \bigwedge^q \Omega_X = \Omega_X^q$ and rewrite over Chern classes of tangent bundle $c(T_X) = \sum_i c_i(T_X) = \prod_i (1 + \alpha_i)$:

$$\begin{aligned} \text{Td}(T_X) &= \prod_i \frac{\alpha_i}{1 - e^{-\alpha_i}} = \sum \text{td}_i, \\ \text{Ch}(\Omega_X^q) &= \sum_{i_1 < i_2 < \dots < i_q} e^{\alpha_{i_1}} \dots e^{\alpha_{i_q}} = \sum \text{ch}_i. \end{aligned} \quad (2)$$

Here α_i are the Chern roots of $T_{\mathbb{P}^n}$.

For the first time the classes $\text{td}(T_X)$ up to td_6 were obtained in [3] (p.14). In the original paper [4] (p.221) Eq.(61) coincides with the td_6 in our notation. All classes up to td_9 were obtained in [5], up to td_{10} in [2].

The Taylor series for (2) is

$$\text{ch}_m(\Omega_n^p) = \sum_{s=1}^{\#I} A_{I_s} X^{I_s}, \quad (3)$$

where

$$A_{I_s} = \frac{1}{I_s!} \binom{n - \#I_s}{p - \#I_s}, \quad X^{I_s} = \sum_k^n \bigwedge^{\#I_s} x_k^{I_s},$$

the multi-index

$$I = \{m, (m-1, 1), (m-2, 2), \dots, (2, \underbrace{1, \dots, 1}_{m-1}), (\underbrace{1, 1, \dots, 1}_m)\}$$

is a tuple of a partition of integer m and $\#I$ is length of a tuple I .

For Chern characters of n -folds X from (2) we have the following theorems.

Theorem 1.

$$\text{ch}_0(\Omega_n^p) = \binom{n}{p}.$$

Theorem 2.

$$\text{ch}_1(\Omega_n^p) = -\binom{n-1}{p-1} c_1.$$

Theorem 3.

$$\text{ch}_2(\Omega_n^p) = \frac{1}{2} \left(\binom{n-1}{p-1} c_1^2 - 2 \binom{n-2}{p-1} c_2 \right).$$

Proof. Since $m = 2$ and $I = \{2, (1, 1)\}$, then we have the following expansion.

For $s = 1$: $I_1 = 2, \#I_1 = 1$,

$$A_{I_1} = A_2 = \frac{1}{2!} \binom{n - \#I_1}{p - \#I_1} = \frac{1}{2} \binom{n - 1}{p - 1},$$

$$X^{I_1} = \sum_k^n \bigwedge^{\#I_1} x_k^{I_1} = \sum_{k=1}^n \bigwedge^1 x_k^2 = \sum_{k=1}^n x_k^2.$$

For $s = 2$: $I_2 = (1, 1), \#I_2 = 2$,

$$A_{I_2} = A_{1,1} = \frac{1}{1!1!} \binom{n - \#I_2}{p - \#I_2} = \binom{n - 2}{p - 2},$$

$$X^{I_2} = \sum_k^n \bigwedge^{\#I_2} x_k^{I_2} = \sum_{k_1 < k_2}^n x_{k_1}^1 x_{k_2}^1.$$

Taking into account

$$\sum x^2 = c_1^2 - 2c_2, \quad \sum x_{k_1}^1 \wedge x_{k_2}^1 = c_2,$$

for (3) we obtain

$$\text{ch}_2(\Omega_n^p) = \sum_{s=1}^2 A_{I_s} X^{I_s} = \frac{1}{2} \binom{n-1}{p-1} (c_1^2 - 2c_2) + \binom{n-2}{p-2} c_2.$$

□

Theorem 4.

$$\text{ch}_3(\Omega_n^p) = -\frac{1}{3!} \left[\binom{n-1}{p-1} c_1^3 - 3 \binom{n-2}{p-1} \left(c_1 c_2 - \frac{n-2p}{n-2} c_3 \right) \right]. \quad (4)$$

Proof. Since $m = 3, I = \{3, (2, 1), (1, 1, 1)\}, \#I = 3$, then we have the following expansion.

For $s = 1$: $I_1 = 3, \#I_1 = 1$,

$$A_{I_1} = A_3 = \frac{1}{3!} \binom{n - \#I_1}{p - \#I_1} = \frac{1}{3!} \binom{n - 1}{p - 1},$$

$$X^{I_1} = \sum_k^n \bigwedge^{\#I_1} x_k^{I_1} = \sum_{k=1}^n \bigwedge^1 x_k^3 = \sum_{k=1}^n x_k^3.$$

For $s = 2$: $I_2 = (2, 1)$, $\#I_1 = 2$,

$$A_{I_2} = A_{2,1} = \frac{1}{2!1!} \binom{n - \#I_2}{p - \#I_2} = \binom{n-2}{p-2},$$

$$X^{I_2} = \sum_k^n \bigwedge^{\#I_2} x_k^{I_2} = \sum_{k_1 < k_2}^n x_{k_1}^2 x_{k_2}^1.$$

For $s = 3$: $I_3 = (1, 1, 1)$, $\#I_1 = 3$,

$$A_{I_3} = A_{1,1,1} = \binom{n - \#I_3}{p - \#I_3} = \binom{n-3}{p-3},$$

$$X^{I_3} = \sum_k^n \bigwedge^{\#I_3} x_k^{I_3} = \sum_{k_1 < k_2 < k_3}^n x_{k_1}^1 x_{k_2}^1 x_{k_3}^1.$$

Taking into account

$$\sum x^3 = c_1^3 - 3c_1 c_2 + 3c_3, \quad \sum x_{k_1}^2 \wedge x_{k_2}^1 = c_1 c_2 - 3c_3,$$

$$\sum x_{k_1}^1 \wedge x_{k_2}^1 \wedge x_{k_3}^1 = c_3,$$

for (3) we obtain

$$\begin{aligned} \text{ch}_3(\Omega_n^p) &= \sum_{s=1}^3 A_{I_s} X^{I_s} \\ &= -\frac{1}{3!} \left[\binom{n-1}{p-1} c_1^3 + 3 \left[-\binom{n-1}{p-1} + \binom{n-2}{p-2} \right] c_1 c_2 \right. \\ &\quad \left. + 3 \left[\binom{n-1}{p-1} - 3 \binom{n-2}{p-2} + 2 \binom{n-3}{p-3} \right] c_3 \right]. \end{aligned}$$

□

Theorem 5.

$$\text{ch}_4(\Omega_n^p) = \frac{1}{4!} \left[\binom{n-1}{p-1} c_1^4 + 4c_2 c_1^2 \left[\binom{n-2}{p-2} - \binom{n-1}{p-1} \right] \right]$$

$$\begin{aligned}
& + 4c_3c_1 \left[3 \binom{n-3}{p-3} - 4 \binom{n-2}{p-2} + \binom{n-1}{p-1} \right] \\
& + 2c_2^2 \left[- \binom{n-2}{p-2} + \binom{n-1}{p-1} \right] \\
& + 4c_4 \left[6 \binom{n-4}{p-4} - 12 \binom{n-3}{p-3} + 7 \binom{n-2}{p-2} - \binom{n-1}{p-1} \right].
\end{aligned} \tag{5}$$

Proof. Let the multi-index $I = \{4, (3, 1), (2, 2), (2, 1, 1), (1, 1, 1, 1)\}$ is a tuple of a integer partition. Since $m = 4$ then we have the following expansion.

$$\begin{aligned}
A_{I_1} X^{I_1} &= \frac{1}{4!} \binom{n-1}{p-1} \sum_{k=1}^n x_k^4 \\
&= \frac{1}{4!} \binom{n-1}{p-1} (c_1^4 - 4c_2c_1^2 + 4c_3c_1 + 2c_2^2 - 4c_4), \\
A_{I_2} X^{I_2} &= \frac{1}{3!1!} \binom{n-2}{p-2} \sum_{k_1 < k_2}^n x_{k_1}^3 x_{k_2}^1 \\
&= \frac{1}{3!} \binom{n-2}{p-2} (c_2c_1^2 - c_3c_1 - 2c_2^2 + 4c_4), \\
A_{I_3} X^{I_3} &= \frac{1}{2!2!} \binom{n-2}{p-2} \sum_{k_1 < k_2}^n x_{k_1}^2 x_{k_2}^2 = \frac{1}{4} \binom{n-2}{p-2} (c_2^2 - 2c_1c_3 + 2c_4), \\
A_{I_4} X^{I_4} &= \frac{1}{2!1!1!} \binom{n-3}{p-3} \sum_{k_1 < k_2 < k_3}^n x_{k_1}^2 x_{k_2}^1 x_{k_3}^1 \\
&= \frac{1}{2!} \binom{n-3}{p-3} (c_1c_3 - 4c_4), \\
A_{I_5} X^{I_5} &= \frac{1}{1!1!1!1!} \binom{n-4}{p-4} \sum_{k_1 < k_2 < k_3 < k_4}^n x_{k_1}^1 x_{k_2}^1 x_{k_3}^1 x_{k_4}^1 = \binom{n-4}{p-4} c_4.
\end{aligned}$$

Substituting this into (3), we obtain (5).

□

Taking into account that h_{ij} is symmetric, we do not write out the highest values for ch_i . The complete formulas for $\text{ch}_m(\Omega_9^p)$ are given in Appendix.

3. Hodge diamond

The Hodge diamond of 2-fold Calabi-Yau can be seen in [1] (p.590), for 3-folds in [6] (p.45). Four-folds were considered in [7], five-folds in [8], six-folds in [2], 7-folds in [9] and 8-folds in [10].

For example, we find the Hodge diamond of 9-fold Calabi-Yau $X_9 \in \mathbb{P}^{13}$, which is a complete intersection of quadrics, cubic, tetric and quintic $X_9 = S^5 \cap S^4 \cap S^3 \cap S^2$:

									1
								0	
						0			1
					0		0		
				0		0		0	1
			0		0		0		
		0		0		0		0	1
	0		0		0		0		
1		10603		1279100		20714272		76456749	1
	0		0		0		0		1
		0		0		0		0	
			0		0		0		1
				0		0		0	
					0		0		1
						0		0	
							0		1
								0	
									1

Summing the Betti numbers
 $\mathbf{b} = (1, 0, 1, 0, 1, 0, 1, 0, 1, 196921450, 1, 0, 1, 0, 1, 0, 1, 0, 1),$

we obtain $\chi = -196921440.$ $\triangle$

This result is easily verified using the theory of characteristic classes. For any morphism $X \xrightarrow{f} \mathbb{P}^n$ with uses the bundle

$$0 - T_X - f^*T_{\mathbb{P}^n} - N_f - 0 \tag{6}$$

we find Chern class $c(T_X)$. Defining $c(\mathcal{O}_{\mathbb{P}^n}(d)) = 1 + dt$, for a complete inter-section k hypersurfaces $X = \bigcap_{i=1}^k S_i$ of degree s_i we obtain $c(N_f) = \prod_{i=1}^k (1 + s_i t).$

The Euler sequence dual to (6) has the form

$$0 - \mathcal{O}_{\mathbb{P}^n} - \mathcal{O}_{\mathbb{P}^n}(1)^{\oplus(n+1)} - T_{\mathbb{P}^n} - 0.$$

From this $c(T_{\mathbb{P}^n}) = (1+t)^{n+1}$, hence

$$c(T_X) = \frac{(1+t)^{n+1}}{\prod_{i=1}^k (1+s_i t)}.$$

The Euler characteristic of m -fold is

$$\chi = \int_X c_m(X) = \int_{\mathbb{P}^n} c_{n-m}(N_f) \wedge c_m(T_X) = c_{n-m}(N_f) \cdot c_m(T_X). \quad (7)$$

For the intersection of $X_9 = S^5 \cap S^4 \cap S^3 \cap S^2 \in \mathbb{P}^{13}$ we have

$$\begin{aligned} c(N_f) &= (1+2t)(1+3t)(1+4t)(1+5t) \\ &= 1+14t+71t^2+154t^3+120t^4, \quad \text{i.e.} \quad c_4(N_f) = 120; \\ c(T_X) &= \frac{(1+t)^{14}}{(1+2t)(1+3t)(1+4t)(1+5t)} \\ &\approx 1+20t^2+\dots-1641012t^9+\dots, \Rightarrow c_9(T_X) = -1641012. \end{aligned}$$

Therefore the Euler characteristic from (7) has the same value χ :

$$\chi = \int_X c_9(X) = \int_{\mathbb{P}^{13}} c_4(N_f) \wedge c_9(T_X) = -196921440. \quad \triangle$$

In the following table we show the Hodge numbers of the 9-fold Calabi-Yay that are complete intersections in ordinary projective spaces.

N	$\mathbb{P}^n X_9$	h^{18}	h^{27}	h^{36}	h^{45}	χ
1	10 11	352595	57402764	962039783	3525659402	9090909080
2	11 10 2	277067	44626868	747049304	2738415965	7060738400
3	11 93	155803	24205468	403667119	1480844938	3817746648
4	11 84	75437	11026301	182705381	671140469	1729895168
5	11 75	35969	4911842	80758445	297132848	765678200
5	11 66	24605	3248534	53176438	195829067	504557280
6	12 922	199351	31564822	527415346	1934044117	4986447264
7	12 832	102035	15397317	255972913	939635995	2422216512
8	12 742	45407	6361784	104913524	385764521	994170464
9	12 733	47498	6710408	110762902	407195612	1049432832
10	12 652	22384	2931072	47939890	176576258	454939200
11	12 643	20109	2600910	42493582	156553227	403335648
12	12 553	12476	1533951	24895276	91846201	236575800
13	12 544	9629	1146317	18530645	68422853	176218880
14	13 8222	129239	19923563	331944063	1217963843	3139921408
15	13 7322	59314	8572583	141844813	521197038	1343347488
16	13 6422	24554	3249688	53233510	196011804	505039104
17	13 6332	24657	3269451	53575579	197257773	508254912
18	13 5522	14931	1878186	30564366	112696521	290308000
19	13 5432	10603	1279100	20714272	76456749	196921440
20	13 5333	9728	1160570	18773812	69311414	178511040

N	$\mathbb{P}^n X_9$	h^{18}	h^{27}	h^{36}	h^{45}	χ
21	13 4442	6725	766781	12323765	45560525	117315584
22	13 4433	5621	625445	10021501	37073645	95452416
23	14 72222	74027	10951223	181643619	667096895	1719531520
24	14 63222	30064	4079938	67030948	246663598	635609088
25	14 54222	12446	1533476	24897546	91847096	236581120
26	14 53322	11354	1381967	22411045	82695878	213000480
27	14 44322	6269	708549	11376205	42066805	108315648
28	14 43332	4775	519488	8299754	30723425	79094880
29	14 33333	3151	322426	5107051	18940276	48745800
30	15 622222	36633	5090345	83848057	308380489	794711040
31	15 532222	13227	1642423	26700259	98471615	253655040
32	15 442222	6945	796641	12815265	47368545	121974784
33	15 433222	5114	561992	8991478	33273932	85665024
34	15 333322	3133	320248	5071816	18810199	48410784
35	16 52 ₁ ...2 ₆	15383	1948475	31752047	117043139	301518080
36	16 432 ₁ ...2 ₅	5407	599027	9595543	35500251	91400448
37	16 3332222	3002	304712	4821238	17884556	46027008
38	17 42 ₁ ...2 ₇	5633	627713	10064897	37229057	95854592
39	17 332 ₁ ...2 ₆	2727	272427	4301119	15962739	41078016
40	18 32 ₁ ...2 ₈	2273	220193	3461473	12858785	33085440
41	19 2 ₁ ...2 ₁₀	1601	145601	2267201	8440001	21708800

Appendix

Taylor series coefficients for

$$\mathrm{Ch}(\Omega_9^p) = \sum \mathrm{ch}_i.$$

$\mathbf{ch}(\Omega_9^1)$:

$$\mathrm{ch}_0 = 9; \quad \mathrm{ch}_1 = -c_1; \quad \mathrm{ch}_2 = \frac{1}{2}(c_1^2 - 2c_2);$$

$$\mathrm{ch}_3 = \frac{1}{3!}(-c_1^3 + 3c_1c_2 - 3c_3);$$

$$\mathrm{ch}_4 = \frac{1}{4!}(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4);$$

$$\mathrm{ch}_5 = \frac{1}{5!}(-c_1^5 + 5c_1^3c_2 - 5c_1c_2^2 - 5c_1^2c_3 + 5c_2c_3 + 5c_1c_4 - 5c_5);$$

$$\mathrm{ch}_6 = \frac{1}{6!} \begin{pmatrix} c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 \\ + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6 \end{pmatrix};$$

$$\mathrm{ch}_7 = \frac{1}{7!} \begin{pmatrix} -c_1^7 + 7c_1^5c_2 - 14c_1^3c_2^2 + 7c_1c_2^3 - 7c_1^4c_3 + 21c_1^2c_2c_3 \\ - 7c_2^2c_3 - 7c_1c_3^2 + 7c_1^3c_4 \\ - 14c_1c_2c_4 + 7c_3c_4 - 7c_1^2c_5 + 7c_2c_5 + 7c_1c_6 - 7c_7 \end{pmatrix};$$

$$\mathrm{ch}_8 = \frac{1}{8!} \begin{pmatrix} c_1^8 - 8c_1^6c_2 + 20c_1^4c_2^2 - 16c_1^2c_2^3 + 2c_2^4 + 8c_1^5c_3 \\ - 32c_1^3c_2c_3 + 24c_1c_2^2c_3 + 12c_1^2c_3^2 - 8c_2c_3^2 - 8c_1^4c_4 \\ + 24c_1^2c_2c_4 - 8c_2^2c_4 - 16c_1c_3c_4 + 4c_4^2 + 8c_1^3c_5 \\ - 16c_1c_2c_5 + 8c_3c_5 - 8c_1^2c_6 + 8c_2c_6 + 8c_1c_7 - 8c_8 \end{pmatrix};$$

$$\mathrm{ch}_9 = \frac{1}{9!} \begin{pmatrix} -c_1^9 + 9c_2c_1^7 - 9c_3c_1^6 - 27c_2^2c_1^5 + 9c_4c_1^5 + 45c_2c_3c_1^4 \\ - 9c_5c_1^4 + 30c_2^3c_1^3 - 18c_3^2c_1^3 - 36c_2c_4c_1^3 + 9c_6c_1^3 \\ - 54c_2^2c_3c_1^2 + 27c_3c_4c_1^2 + 27c_2c_5c_1^2 - 9c_7c_1^2 - 9c_2^4c_1 \\ + 27c_2^2c_3^2c_1 - 9c_4^2c_1 + 27c_2^2c_4c_1 - 18c_3c_5c_1 \\ - 18c_2c_6c_1 + 9c_8c_1 - 3c_3^3 + 9c_3^2c_3 - 18c_2c_3c_4 \\ - 9c_2^2c_5 + 9c_4c_5 + 9c_3c_6 + 9c_2c_7 - 9c_9 \end{pmatrix}.$$

$\mathbf{ch}(\Omega_9^2)$:

$$\mathrm{ch}_0 = 36; \quad \mathrm{ch}_1 = -8c_1; \quad \mathrm{ch}_2 = 4c_1^2 - 7c_2;$$

$$\mathrm{ch}_3 = \frac{1}{3!}(-8c_1^3 + 21c_2c_1 - 15c_3);$$

$$\mathrm{ch}_4 = \frac{2}{4!}(4c_1^4 - 14c_2c_1^2 + 8c_3c_1 + 7c_2^2 - 2c_4);$$

$$\mathrm{ch}_5 = \frac{1}{5!} \begin{pmatrix} -8c_1^5 + 35c_2c_1^3 - 15c_3c_1^2 \\ - 35c_2^2c_1 - 15c_4c_1 + 25c_2c_3 + 35c_5 \end{pmatrix};$$

$$\mathrm{ch}_6 = \frac{1}{6!} \begin{pmatrix} 8c_1^6 - 42c_2c_1^4 + 12c_3c_1^3 + 63c_2^2c_1^2 - 54c_2c_3c_1 \\ 48c_4c_1^2 - 108c_5c_1 - 14c_2^3 + 21c_3^2 - 18c_2c_4 + 138c_6 \end{pmatrix};$$

$$\begin{aligned}
\text{ch}_7 &= \frac{1}{7!} \begin{pmatrix} -8c_1^7 + 49c_2c_1^5 - 7c_3c_1^4 - 98c_2^2c_1^3 + 84c_2c_3c_1^2 \\ -98c_4c_1^3 + 238c_5c_1^2 + 49c_2^3c_1 - 56c_2^2c_1 + 98c_2c_4c_1 \\ -343c_6c_1 - 35c_2^2c_3 + 35c_3c_4 - 175c_2c_5 + 385c_7 \end{pmatrix}; \\
\text{ch}_8 &= \frac{2}{8!} \begin{pmatrix} 4c_1^8 - 28c_2c_1^6 + 70c_2^2c_1^4 + 84c_4c_1^4 - 56c_2c_3c_1^3 \\ -224c_5c_1^3 - 56c_2^3c_1^2 + 56c_2^2c_1^2 - 140c_2c_4c_1^2 \\ +364c_6c_1^2 + 56c_2^2c_3c_1 - 56c_3c_4c_1 + 336c_2c_5c_1 \\ -448c_7c_1 + 7c_2^4 - 28c_2c_3^2 + 42c_4^2 + 28c_2^2c_4 \\ -56c_3c_5 - 308c_2c_6 + 476c_8 \end{pmatrix}; \\
\text{ch}_9 &= \frac{1}{9!} \begin{pmatrix} -8c_1^9 + 63c_2c_1^7 + 9c_3c_1^6 - 189c_2^2c_1^5 - 15c_3^3 + 45c_2^3c_3 \\ -261c_4c_1^5 + 135c_2c_3c_1^4 + 765c_5c_1^4 + 210c_2^3c_1^3 \\ -198c_2^2c_1^3 + 612c_2c_4c_1^3 - 1395c_6c_1^3 - 234c_2^2c_3c_1^2 \\ +279c_3c_4c_1^2 - 1737c_2c_5c_1^2 + 1899c_7c_1^2 - 63c_2^4c_1 \\ +207c_2c_3^2c_1 - 333c_4^2c_1 - 297c_2^2c_4c_1 + 342c_3c_5c_1 \\ +2358c_2c_6c_1 - 2151c_8c_1 - 90c_2c_3c_4 - 711c_3c_6 \\ +459c_2^2c_5 + 297c_4c_5 - 1719c_2c_7 + 2223c_9 \end{pmatrix}.
\end{aligned}$$

ch(Ω_9^3):

$$\text{ch}_0 = 84; \quad \text{ch}_1 = -28c_1; \quad \text{ch}_2 = 7(2c_1^2 - 3c_2);$$

$$\text{ch}_3 = \frac{1}{6}(-28c_1^3 + 63c_2c_1 - 27c_3);$$

$$\text{ch}_4 = \frac{1}{12}(14c_1^4 - 42c_2c_1^2 + 6c_3c_1 + 21c_2^2 + 18c_4);$$

$$\text{ch}_5 = \frac{1}{120} \begin{pmatrix} -28c_1^5 + 105c_2c_1^3 + 15c_3c_1^2 \\ -105c_2^2c_1 - 135c_4c_1 + 45c_2c_3 + 135c_5 \end{pmatrix};$$

$$\text{ch}_6 = \frac{1}{720} \begin{pmatrix} 28c_1^6 - 126c_2c_1^4 - 54c_3c_1^3 + 189c_2^2c_1^2 \\ +294c_4c_1^2 - 72c_2c_3c_1 - 294c_5c_1 - 42c_2^3 \\ +63c_3^2 - 174c_2c_4 + 54c_6 \end{pmatrix};$$

$$\text{ch}_7 = \frac{1}{720} \begin{pmatrix} -4c_1^7 + 21c_2c_1^5 + 15c_3c_1^4 - 42c_2^2c_1^3 - 75c_4c_1^3 \\ +9c_2c_3c_1^2 + 75c_5c_1^2 + 21c_2^3c_1 - 27c_2^2c_1 + 96c_2c_4c_1 \\ +45c_6c_1 - 9c_2^2c_3 + 9c_3c_4 - 81c_2c_5 - 189c_7 \end{pmatrix};$$

$$\text{ch}_8 = \frac{1}{2880} \begin{pmatrix} 2c_1^8 - 12c_2c_1^6 - 12c_3c_1^5 + 30c_2^2c_1^4 \\ +60c_4c_1^4 - 60c_5c_1^3 - 24c_2^3c_1^2 + 30c_2^2c_1^2 \\ -120c_2c_4c_1^2 - 100c_6c_1^2 + 12c_2^2c_3c_1 - 20c_3c_4c_1 \\ +140c_2c_5c_1 + 388c_7c_1 + 3c_2^4 - 12c_2c_3^2 \\ +26c_4^2 + 28c_2^2c_4 - 48c_3c_5 + 12c_2c_6 - 612c_8 \end{pmatrix};$$

$$\text{ch}_9 = \frac{1}{9!} \begin{pmatrix} -28c_1^9 + 189c_2c_1^7 + 243c_3c_1^6 - 567c_2^2c_1^5 \\ -1251c_4c_1^5 - 135c_2c_3c_1^4 + 1251c_5c_1^4 + 630c_2^3c_1^3 \\ -810c_2^3c_1^3 + 3420c_2c_4c_1^3 + 3789c_6c_1^3 - 270c_2^2c_3c_1^2 \\ +855c_3c_4c_1^2 - 4689c_2c_5c_1^2 - 15885c_7c_1^2 - 189c_2^4c_1 \\ +675c_2c_3^2c_1 - 1557c_4^2c_1 - 1845c_2^2c_4c_1 \\ +2430c_3c_5c_1 - 2106c_2c_6c_1 + 29997c_8c_1 - 27c_3^3 \\ +81c_2^3c_3 - 162c_2c_3c_4 + 1431c_2^2c_5 + 837c_4c_5 \\ -2187c_3c_6 + 9909c_2c_7 - 38637c_9 \end{pmatrix}.$$

$\text{ch}(\Omega_9^4)$:

$$\text{ch}_0 = 126; \quad \text{ch}_1 = -56c_1; \quad \text{ch}_2 = 7(4c_1^2 - 5c_2);$$

$$\text{ch}_3 = \frac{1}{6}(-56c_1^3 + 105c_2c_1 - 15c_3);$$

$$\text{ch}_4 = \frac{1}{12}(28c_1^4 - 70c_2c_1^2 - 20c_3c_1 + 35c_2^2 + 50c_4);$$

$$\text{ch}_5 = \frac{1}{120} \begin{pmatrix} -56c_1^5 + 175c_2c_1^3 + 125c_3c_1^2 \\ -175c_2^2c_1 - 275c_4c_1 + 25c_2c_3 + 95c_5 \end{pmatrix};$$

$$\text{ch}_6 = \frac{1}{6!} \begin{pmatrix} 56c_1^6 - 210c_2c_1^4 - 240c_3c_1^3 + 315c_2^2c_1^2 + 540c_4c_1^2 \\ +30c_2c_3c_1 - 70c_2^3 + 105c_3^2 - 390c_2c_4 - 570c_6 \end{pmatrix};$$

$$\text{ch}_7 = \frac{1}{6!} \begin{pmatrix} -8c_1^7 + 35c_2c_1^5 + 55c_3c_1^4 - 70c_2^2c_1^3 \\ -130c_4c_1^3 - 30c_2c_3c_1^2 - 50c_5c_1^2 + 35c_2^3c_1 \\ -50c_2^3c_1 + 200c_2c_4c_1 + 335c_6c_1 - 5c_2^2c_3 \\ +5c_3c_4 - 55c_2c_5 - 245c_7 \end{pmatrix};$$

$$\text{ch}_8 = \frac{1}{2880} \begin{pmatrix} 4c_1^8 - 20c_2c_1^6 - 40c_3c_1^5 + 50c_2^2c_1^4 \\ +100c_4c_1^4 + 40c_2c_3c_1^3 + 80c_5c_1^3 - 40c_2^3c_1^2 \\ +60c_2^3c_1^2 - 240c_2c_4c_1^2 - 460c_6c_1^2 - 20c_3c_4c_1 \\ +60c_2c_5c_1 + 280c_7c_1 + 5c_2^4 - 20c_2c_3^2 \\ +50c_2^2c_4 - 100c_3c_5 + 260c_2c_6 + 700c_8 \end{pmatrix};$$

$$\text{ch}_9 = \frac{1}{9!} \begin{pmatrix} -56c_1^9 + 315c_2c_1^7 + 765c_3c_1^6 - 945c_2^2c_1^5 \\ -2025c_4c_1^5 - 1125c_2c_3c_1^4 - 2511c_5c_1^4 + 1050c_2^3c_1^3 \\ -1710c_2^3c_1^3 + 6660c_2c_4c_1^3 + 14481c_6c_1^3 + 270c_2^2c_3c_1^2 \\ +1035c_3c_4c_1^2 - 981c_2c_5c_1^2 - 6921c_7c_1^2 - 315c_2^4c_1 \\ +1215c_2c_3^2c_1 - 3105c_4^2c_1 - 3825c_2^2c_4c_1 \\ +5886c_3c_5c_1 - 17298c_2c_6c_1 - 54819c_8c_1 - 15c_3^3 \\ +45c_2^3c_3 - 90c_2c_3c_4 + 963c_2^2c_5 + 549c_4c_5 \\ -1467c_3c_6 + 11637c_2c_7 + 140571c_9 \end{pmatrix}.$$

$\text{ch}(\Omega_9^5)$:

$$\begin{aligned}
\text{ch}_0 &= 126; \quad \text{ch}_1 = -70c_1; \quad \text{ch}_2 = 35(c_1^2 - c_2); \\
\text{ch}_3 &= \frac{5}{6}(-14c_1^3 + 21c_2c_1 + 3c_3); \\
\text{ch}_4 &= \frac{5}{12}(7c_1^4 - 14c_2c_1^2 - 10c_3c_1 + 7c_2^2 + 10c_4); \\
\text{ch}_5 &= \frac{1}{24} \begin{pmatrix} -14c_1^5 + 35c_2c_1^3 \\ +45c_3c_1^2 - 35c_2^2c_1 - 45c_4c_1 - 5c_2c_3 - 19c_5 \end{pmatrix}; \\
\text{ch}_6 &= \frac{1}{144} \begin{pmatrix} 14c_1^6 - 42c_2c_1^4 - 78c_3c_1^3 + 63c_2^2c_1^2 \\ +78c_4c_1^2 + 36c_2c_3c_1 + 114c_5c_1 - 14c_2^3 \\ +21c_3^2 - 78c_2c_4 - 114c_6 \end{pmatrix}; \\
\text{ch}_7 &= \frac{1}{144} \begin{pmatrix} -2c_1^7 + 7c_2c_1^5 + 17c_3c_1^4 - 14c_2^2c_1^3 - 17c_4c_1^3 \\ -15c_2c_3c_1^2 - 47c_5c_1^2 + 7c_2^3c_1 - 11c_3^2c_1 + 38c_2c_4c_1 \\ +47c_6c_1 + c_2^2c_3 - c_3c_4 + 11c_2c_5 + 49c_7 \end{pmatrix}; \\
\text{ch}_8 &= \frac{1}{576} \begin{pmatrix} c_1^8 - 4c_2c_1^6 - 12c_3c_1^5 + 10c_2^2c_1^4 \\ +12c_4c_1^4 + 16c_2c_3c_1^3 + 52c_5c_1^3 - 8c_2^3c_1^2 \\ +14c_3^2c_1^2 - 44c_2c_4c_1^2 - 52c_6c_1^2 - 4c_2^2c_3c_1 \\ -32c_2c_5c_1 - 140c_7c_1 + c_2^4 - 4c_2c_3^2 \\ +10c_4^2 + 12c_2^2c_4 - 20c_3c_5 + 52c_2c_6 + 140c_8 \end{pmatrix}; \\
\text{ch}_9 &= \frac{1}{9!} \begin{pmatrix} -70c_1^9 + 315c_2c_1^7 + 1125c_3c_1^6 - 945c_2^2c_1^5 \\ -1125c_4c_1^5 - 2025c_2c_3c_1^4 - 6939c_5c_1^4 + 1050c_2^3c_1^3 \\ -2070c_3^2c_1^3 + 5940c_2c_4c_1^3 + 6939c_6c_1^3 + 990c_2^2c_3c_1^2 \\ 225c_3c_4c_1^2 + 7281c_2c_5c_1^2 + 33381c_7c_1^2 - 315c_2^4c_1 \\ 1305c_2c_3^2c_1 - 3195c_4^2c_1 - 3735c_2^2c_4c_1 + 6714c_3c_5c_1 \\ -15462c_2c_6c_1 - 33381c_8c_1 + 15c_3^3 - 45c_2^3c_3 \\ +90c_2c_3c_4 - 963c_2^2c_5 - 549c_4c_5 + 1467c_3c_6 \\ -11637c_2c_7 - 140571c_9 \end{pmatrix}.
\end{aligned}$$

ch(Ω_9^6):

$$\begin{aligned}
\text{ch}_0 &= 84; \quad \text{ch}_1 = -56c_1; \quad \text{ch}_2 = 7(4c_1^2 - 3c_2); \\
\text{ch}_3 &= \frac{1}{6}(-56c_1^3 + 63c_2c_1 + 27c_3); \\
\text{ch}_4 &= \frac{1}{12}(28c_1^4 - 42c_2c_1^2 - 48c_3c_1 + 21c_2^2 + 18c_4); \\
\text{ch}_5 &= \frac{1}{120} \begin{pmatrix} -56c_1^5 + 105c_2c_1^3 + 195c_3c_1^2 \\ -105c_2^2c_1 - 45c_4c_1 - 45c_2c_3 - 135c_5 \end{pmatrix}; \\
\text{ch}_6 &= \frac{1}{720} \begin{pmatrix} 56c_1^6 - 126c_2c_1^4 - 324c_3c_1^3 + 189c_2^2c_1^2 \\ +24c_4c_1^2 + 198c_2c_3c_1 + 516c_5c_1 - 42c_2^3 \\ +63c_3^2 - 174c_2c_4 + 54c_6 \end{pmatrix};
\end{aligned}$$

$$\begin{aligned}
\text{ch}_7 &= \frac{1}{720} \begin{pmatrix} -8c_1^7 + 21c_2c_1^5 + 69c_3c_1^4 - 42c_2^2c_1^3 \\ +6c_4c_1^3 - 72c_2c_3c_1^2 - 186c_5c_1^2 + 21c_2^3c_1 \\ -36c_3^2c_1 + 78c_2c_4c_1 - 99c_6c_1 + 9c_2^2c_3 \\ -9c_3c_4 + 81c_2c_5 + 189c_7 \end{pmatrix}; \\
\text{ch}_8 &= \frac{1}{2880} \begin{pmatrix} 4c_1^8 - 12c_2c_1^6 - 48c_3c_1^5 + 30c_2^2c_1^4 - 12c_4c_1^4 \\ +72c_2c_3c_1^3 + 192c_5c_1^3 - 24c_2^3c_1^2 + 48c_3^2c_1^2 \\ -84c_2c_4c_1^2 + 188c_6c_1^2 - 24c_2^2c_3c_1 + 16c_3c_4c_1 \\ -184c_2c_5c_1 - 368c_7c_1 + 3c_2^4 - 12c_2c_3^2 \\ +26c_4^2 + 28c_2^2c_4 - 48c_3c_5 + 12c_2c_6 - 612c_8 \end{pmatrix}; \\
\text{ch}_9 &= \frac{1}{9!} \begin{pmatrix} -56c_1^9 + 189c_2c_1^7 + 891c_3c_1^6 - 567c_2^2c_1^5 \\ +369c_4c_1^5 - 1755c_2c_3c_1^4 - 4905c_5c_1^4 + 630c_2^3c_1^3 \\ -1458c_2^2c_3^2 + 2124c_2c_4c_1^3 - 7065c_6c_1^3 + 1026c_2^2c_3c_1^2 \\ -603c_3c_4c_1^2 + 7461c_2c_5c_1^2 + 14625c_7c_1^2 - 189c_2^4c_1 \\ +837c_2c_3^2c_1 - 1719c_4^2c_1 - 1683c_2^2c_4c_1 \\ +594c_2c_6c_1 + 47115c_8c_1 + 27c_3^3 - 81c_2^3c_3 \\ +162c_2c_3c_4 - 1431c_2^2c_5 - 837c_4c_5 + 2187c_3c_6 \\ +3618c_3c_5c_1 - 9909c_2c_7 + 38637c_9 \end{pmatrix}.
\end{aligned}$$

$\text{ch}(\Omega_9^7)$:

$$\text{ch}_0 = 36; \text{ch}_1 = 28c_1; \text{ch}_2 = 7(2c_1^2 - c_2);$$

$$\text{ch}_3 = \frac{1}{6}(-28c_1^3 + 21c_2c_1 + 15c_3);$$

$$\text{ch}_4 = \frac{1}{12}(14c_1^4 - 14c_2c_1^2 - 22c_3c_1 + 7c_2^2 - 2c_4);$$

$$\text{ch}_5 = \frac{1}{120} \begin{pmatrix} -28c_1^5 + 35c_2c_1^3 + 85c_3c_1^2 \\ -35c_2^2c_1 + 35c_4c_1 - 25c_2c_3 - 35c_5 \end{pmatrix};$$

$$\text{ch}_6 = \frac{1}{720} \begin{pmatrix} 28c_1^6 - 42c_2c_1^4 - 138c_3c_1^3 + 63c_2^2c_1^2 \\ -102c_4c_1^2 + 96c_2c_3c_1 + 102c_5c_1 - 14c_2^3 \\ +21c_3^2 - 18c_2c_4 + 138c_6 \end{pmatrix};$$

$$\text{ch}_7 = \frac{1}{720} \begin{pmatrix} -4c_1^7 + 7c_2c_1^5 + 29c_3c_1^4 - 14c_2^2c_1^3 + 31c_4c_1^3 \\ -33c_2c_3c_1^2 - 31c_5c_1^2 + 7c_2^3c_1 - 13c_3^2c_1 + 4c_2c_4c_1 \\ -89c_6c_1 + 5c_2^2c_3 - 5c_3c_4 + 25c_2c_5 - 55c_7 \end{pmatrix};$$

$$\text{ch}_8 = \frac{1}{2880} \begin{pmatrix} 2c_1^8 - 4c_2c_1^6 - 20c_3c_1^5 + 10c_2^2c_1^4 - 28c_4c_1^4 \\ +32c_2c_3c_1^3 + 28c_5c_1^3 - 8c_2^3c_1^2 + 18c_3^2c_1^2 + 132c_6c_1^2 \\ -12c_2^2c_3c_1 + 12c_3c_4c_1 - 52c_2c_5c_1 + 156c_7c_1 + c_2^4 \\ -4c_2c_3^2 + 6c_4^2 + 4c_2^2c_4 - 8c_3c_5 - 44c_2c_6 + 68c_8 \end{pmatrix};$$

$$\text{ch}_9 = \frac{1}{9!} \begin{pmatrix} -28c_1^9 + 63c_2c_1^7 + 369c_3c_1^6 - 189c_2^2c_1^5 \\ +639c_4c_1^5 - 765c_2c_3c_1^4 - 639c_5c_1^4 + 210c_2^3c_1^3 \\ -558c_3^2c_1^3 - 108c_2c_4c_1^3 - 4401c_6c_1^3 + 486c_2^2c_3c_1^2 \\ -531c_3c_4c_1^2 + 1989c_2c_5c_1^2 - 7695c_7c_1^2 - 63c_2^4c_1 \\ +297c_2c_3^2c_1 - 423c_4^2c_1 - 207c_2^2c_4c_1 + 666c_3c_5c_1 \\ +3186c_2c_6c_1 - 6417c_8c_1 + 15c_3^3 - 45c_2^3c_3 \\ +90c_2c_3c_4 - 459c_2^2c_5 - 297c_4c_5 \\ +711c_3c_6 + 1719c_2c_7 - 2223c_9 \end{pmatrix}.$$

ch(Ω_9^8):

$$\text{ch}_0 = 9; \quad \text{ch}_1 = -8c_1; \quad \text{ch}_2 = 4c_1^2 - c_2;$$

$$\text{ch}_3 = \frac{1}{6} (-8c_1^3 + 3c_2c_1 + 3c_3);$$

$$\text{ch}_4 = \frac{1}{12} (4c_1^4 - 2c_2c_1^2 - 4c_3c_1 + c_2^2 - 2c_4);$$

$$\text{ch}_5 = \frac{1}{120} (-8c_1^5 + 5c_2c_1^3 + 15c_3c_1^2 - 5c_2^2c_1 + 15c_4c_1 - 5c_2c_3 + 5c_5);$$

$$\text{ch}_6 = \frac{1}{720} \begin{pmatrix} 8c_1^6 - 6c_2c_1^4 - 24c_3c_1^3 + 9c_2^2c_1^2 - 36c_4c_1^2 \\ +18c_2c_3c_1 - 24c_5c_1 - 2c_2^3 + 3c_3^2 + 6c_2c_4 - 6c_6 \end{pmatrix};$$

$$\text{ch}_7 = \frac{1}{5040} \begin{pmatrix} -8c_1^7 + 7c_2c_1^5 + 35c_3c_1^4 - 14c_2^2c_1^3 + 70c_4c_1^3 \\ -42c_2c_3c_1^2 + 70c_5c_1^2 + 7c_2^3c_1 - 14c_3^2c_1 - 28c_2c_4c_1 \\ +35c_6c_1 + 7c_2^2c_3 - 7c_3c_4 - 7c_2c_5 + 7c_7 \end{pmatrix};$$

$$\text{ch}_8 = \frac{2}{8!} \begin{pmatrix} 4c_1^8 - 4c_2c_1^6 - 24c_3c_1^5 + 10c_2^2c_1^4 \\ -60c_4c_1^4 + 40c_2c_3c_1^3 - 80c_5c_1^3 - 8c_2^3c_1^2 \\ +20c_3^2c_1^2 + 40c_2c_4c_1^2 - 60c_6c_1^2 - 16c_2^2c_3c_1 \\ +20c_3c_4c_1 + 20c_2c_5c_1 - 24c_7c_1 + c_2^4 - 4c_2c_3^2 \\ +2c_4^2 - 4c_2^2c_4 + 4c_3c_5 + 4c_2c_6 - 4c_8 \end{pmatrix};$$

$$\text{ch}_9 = \frac{1}{9!} \begin{pmatrix} -8c_1^9 + 9c_2c_1^7 + 63c_3c_1^6 - 27c_2^2c_1^5 + 189c_4c_1^5 \\ -135c_2c_3c_1^4 + 315c_5c_1^4 + 30c_2^3c_1^3 - 90c_3^2c_1^3 \\ -180c_2c_4c_1^3 + 315c_6c_1^3 + 90c_2^2c_3c_1^2 - 135c_3c_4c_1^2 \\ -135c_2c_5c_1^2 + 189c_7c_1^2 - 9c_2^4c_1 + 45c_2c_3^2c_1 \\ -27c_4^2c_1 + 45c_2^2c_4c_1 - 54c_3c_5c_1 - 54c_2c_6c_1 \\ +63c_8c_1 + 3c_3^3 - 9c_2^3c_3 + 18c_2c_3c_4 \\ +9c_2^2c_5 - 9c_4c_5 - 9c_3c_6 - 9c_2c_7 + 9c_9 \end{pmatrix}.$$

ch(Ω_9^9):

$$\text{ch}_k = (-)^k \frac{c_1^k}{k!}.$$

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