MODELING THE PROCESS OF CARGO TRANSPORTATION BETWEEN NODE STATIONS

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Abstract: This article is devoted to the study of the model of the organization of railway cargo transportation on the section of the railway network between two node stations. The main characteristic of the stations is the inconsistency degree between the reception and dispatch of goods, which is the difference between the volume of incoming and outgoing cargo per unit of time. It is important for the smooth organization of cargo transportation that the specified characteristic has small values over time at all stations. The modes of cargo transportation under which this will be carried out have been identified.

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1. Introduction

Transport is one of the key industries in any state. This determines the great demand for all kinds of research that offers solutions to problems arising in this industry. First of all, we are talking about research on the modeling of traffic flows (see [1], [2], [3], [4], [5], [6], [7]) and the organization of the transportation process (see [8], [9], [10]).

In the papers(see [11], [12], [13], [14], [15]) a model of the organization of cargo transportation on an extended route between two node stations is investigated. It is assumed that there is a special area between the two neighboring

stations where part of the cargo can be temporarily stored in case of high load of stations. The movement of goods occurs in one direction. The organization of cargo traffic is carried out using two technologies. The first technology sets the rules for the interaction of neighboring stations, and the second technology sets the rules for the interaction of stations with neighboring storage areas.

Further research in this direction is given in (see [16]). It presents a model describing the process of cargo transportation between two node stations in the absence of special storage areas between stations. This article describes the modification of this model. Stations have different technical potentials and may have different degrees of loading. A new characteristic of the stations is introduced - the inconsistency degree between the reception and dispatch of goods. The main task of this research is to study the dynamics of this characteristic depending on the demand for cargo transportation, the technical capabilities of stations, the degree of their loading, as well as to identify the modes of cargo transportation in which this characteristic takes minimum values at all stations.

2. Model description

Consider the movement of goods on a railway line between two node stations connected by a set of intermediate stations. Let the number of intermediate stations be equal to m . Denoting by 0 and $m + 1$, respectively the numbers of the initial and final node stations, we obtain the following set of station numbers: $\{0, 1, \dots, m, m + 1\}$. Denote by n_i , $i = 0, 1, \dots, m + 1$ the number of roads at the station with the number i .

Consider discrete moments of time

$$t_0, t_1, t_2, \dots; \quad t_k = t_{k-1} + \Delta t, k = 1, 2, \dots$$

Let $\bar{V}_{ij}(t_k)$ be the volume of goods received on the j -th road of the i -th station during the time interval $[t_{k-1}, t_k]$, and $\bar{\bar{V}}_{ij}(t_k)$ is the volume of goods sent from the j -th road of the i -th station during the time interval $[t_{k-1}, t_k]$.

Let us denote

$$x_{ij}(t_k) = \begin{cases} \frac{\bar{V}_{ij}(t_k) - \bar{\bar{V}}_{ij}(t_k)}{\bar{V}_{ij}(t_k)}, & \text{if } \bar{V}_{ij}(t_k) > \bar{\bar{V}}_{ij}(t_k), \\ 0, & \text{if } \bar{V}_{ij}(t_k) \leq \bar{\bar{V}}_{ij}(t_k). \end{cases}$$

It is obvious that

$$0 \leq x_{ij}(t_k) \leq 1, \quad i = 0, 1, \dots, m + 1; \quad j = 1, 2, \dots, n_i,$$

and characterize the inconsistency degree between the reception and dispatch of goods on j -th road of the i -th station at the time t_k .

Let us denote by

$$z_i(t_k) = \frac{1}{n_i} \sum_{j=0}^{n_i} x_{ij}(t_k).$$

It is also obvious that $0 \leq z_i(t_k) \leq 1$ and characterizes the inconsistency degree between the reception and dispatch of goods to the i -th station at the time t_k .

The technical potential of an arbitrary station with the number i , $i = 0, 1, \dots, m+1$, is determined by the maximum allowable increase in inconsistency degree between the reception and dispatch of goods at this station per unit of time and is given by a continuous decreasing function $\varphi_i(z)$, defined on the segment $[0, 1]$ and satisfying the condition $\varphi_i(1) = 0$, $i = 0, 1, \dots, m+1$.

The initial node station receives goods depending on the demand for transportation within its technical potential. Sending cargo from an arbitrary station with the number i , $i = 0, 1, \dots, m$ to the next station (with the number $i+1$) is carried out taking into account the volume of cargo at this station and the technical potential of the next station. Note that sending goods from the station with the number i to the next station (with the number $i+1$) is equivalent to receiving goods to the $i+1$ station from the previous station (with the number i). The final node station distributes cargo with a certain intensity taking into account the volume of goods on it. Taking into account the above, let us write down a system of finite-difference equations describing the change in the inconsistency degree between the reception and dispatch of goods at stations:

$$\begin{aligned} z_0(t_k) &= z_0(t_{k-1}) + [\min(d_0, \varphi_0(z_0(t_{k-1}))) \\ &\quad - \min(z_0(t_{k-1}) + \min(d_0, \varphi_0(z_0(t_{k-1}))), \\ &\quad \lambda_1 \varphi_1(z_1(t_{k-1})))] \Delta t, \quad k = 1, 2, 3, \dots, \end{aligned} \quad (1)$$

$$0 \leq z_0(t_k) \leq 1, \quad k = 0, 1, 2, \dots, \quad (2)$$

$$\begin{aligned} z_1(t_k) &= z_1(t_{k-1}) + [\min(z_0(t_{k-1}) + \min(d_0, \varphi_0(z_0(t_{k-1}))), \\ &\quad \lambda_1 \varphi_1(z_1(t_{k-1}))) - \min(z_1(t_{k-1}) + \min(z_0(t_{k-1}) \\ &\quad + \min(d_0, \varphi_0(z_0(t_{k-1}))), \lambda_1 \varphi_1(z_1(t_{k-1}))), \\ &\quad \lambda_2 \varphi_2(z_2(t_{k-1})))] \Delta t, \quad k = 1, 2, 3, \dots, \end{aligned} \quad (3)$$

$$0 \leq z_1(t_k) \leq 1, \quad k = 0, 1, 2, \dots, \quad (4)$$

...

$$z_i(t_k) = z_i(t_{k-1}) + [\min(z_{i-1}(t_{k-1}) + \min(z_{i-2}(t_{k-1})$$

$$\begin{aligned}
& + \dots + \min(z_0(t_{k-1}) + \min(d_0, \varphi_0(z_0(t_{k-1}))), \\
& \lambda_1 \varphi_1(z_1(t_{k-1}))), \lambda_2 \varphi_2(z_2(t_{k-1}))), \dots, \lambda_i \varphi_i(z_i(t_{k-1}))) \\
& - \min(z_i(t_{k-1}) + \min(z_{i-1}(t_{k-1}) + \dots + \min(z_0(t_{k-1}) \\
& + \min(d_0, \varphi_0(z_0(t_{k-1}))), \lambda_1 \varphi_1(z_1(t_{k-1}))), \\
& \lambda_2 \varphi_2(z_2(t_{k-1}))), \dots, \lambda_{i+1} \varphi_{i+1}(z_{i+1}(t_{k-1}))) \Delta t, \\
& k = 1, 2, 3, \dots,
\end{aligned} \tag{5}$$

$$0 \leq z_i(t_k) \leq 1, \quad k = 0, 1, 2, \dots, \tag{6}$$

...

$$\begin{aligned}
z_{m+1}(t_k) = & z_{m+1}(t_{k-1}) + [\min(z_m(t_{k-1}) + \min(z_{m-1}(t_{k-1}) \\
& + \dots + \min(z_0(t_{k-1}) + \min(d_0, \varphi_0(z_0(t_{k-1}))), \\
& \lambda_1 \varphi_1(z_1(t_{k-1}))), \lambda_2 \varphi_2(z_2(t_{k-1}))), \dots, \\
& \lambda_{m+1} \varphi_{m+1}(z_{m+1}(t_{k-1}))) - \min(z_{m+1}(t_{k-1}) \\
& + \min(z_m(t_{k-1}) + \dots + \min(z_0(t_{k-1}) \\
& + \min(d_0, \varphi_0(z_0(t_{k-1}))), \lambda_1 \varphi_1(z_1(t_{k-1}))), \\
& \lambda_2 \varphi_2(z_2(t_{k-1}))), \dots, \lambda_{m+1} \varphi_{m+1}(z_{m+1}(t_{k-1}))), \\
& d_{m+1}] \Delta t, k = 1, 2, 3, \dots,
\end{aligned} \tag{7}$$

$$0 \leq z_{m+1}(t_k) \leq 1, \quad k = 0, 1, 2, \dots, \tag{8}$$

Here $d_0 > 0$, $0 < \lambda_i \leq 1$, $i = 1, 2, \dots, m+1$, d_{m+1} are the parameters of the model:

d_0 is a characteristics of the demand for transportation;

λ_i is a characteristic of the utilization degree of the technical potential of the station with the number i ;

d_{m+1} is a characteristic of the cargo distribution mode from the final node station.

Let us move on to the continuous analog of the system of discrete-difference equations (1) – (8), presented below

$$\begin{aligned}
\dot{z}_0(t) = & \min(d_0, \varphi_0(z_0(t))) \\
& - \min(z_0(t) + \min(d_0, \varphi_0(z_0(t))), \\
& \lambda_1 \varphi_1(z_1(t))), \quad t \in [t_0, +\infty), \\
\dot{z}_1(t) = & \min(z_0(t) + \min(d_0, \varphi_0(z_0(t))), \\
& \lambda_1 \varphi_1(z_1(t))) - \min(z_1(t) + \min(z_0(t) \\
& + \min(d_0, \varphi_0(z_0(t))), \lambda_1 \varphi_1(z_1(t))),
\end{aligned} \tag{9}$$

$$\lambda_2 \varphi_2(z_2(t))), \quad t \in [t_0, +\infty), \quad (10)$$

$$\begin{aligned} & \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \dot{z}_i(t) = & \min(z_{i-1}(t) + \min(z_{i-2}(t) \\ & + \dots + \min(z_0(t) + \min(d_0, \varphi_0(z_0(t))), \\ & \lambda_1 \varphi_1(z_1(t))), \lambda_2 \varphi_2(z_2(t))), \dots, \lambda_i \varphi_i(z_i(t))) \\ & - \min(z_i(t) + \min(z_{i-1}(t) + \dots + \min(z_0(t) \\ & + \min(d_0, \varphi_0(z_0(t))), \lambda_1 \varphi_1(z_1(t))), \\ & \lambda_2 \varphi_2(z_2(t))), \dots, \lambda_{i+1} \varphi_{i+1}(z_{i+1}(t))), \quad t \in [t_0, +\infty), \end{aligned} \quad (11)$$

$$\begin{aligned} & \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ \dot{z}_{m+1}(t) = & \min(z_m(t) + \min(z_{m-1}(t) \\ & + \dots + \min(z_0(t) + \min(d_0, \varphi_0(z_0(t))), \\ & \lambda_1 \varphi_1(z_1(t))), \lambda_2 \varphi_2(z_2(t))), \dots, \\ & \lambda_{m+1} \varphi_{m+1}(z_{m+1}(t))) - \min(z_{m+1}(t) \\ & + \min(z_m(t) + \dots + \min(z_0(t) \\ & + \min(d_0, \varphi_0(z_0(t))), \lambda_1 \varphi_1(z_1(t))), \\ & \lambda_2 \varphi_2(z_2(t))), \dots, \lambda_{m+1} \varphi_{m+1}(z_{m+1}(t))), \\ & d_{m+1}), \quad t \in [t_0, +\infty). \end{aligned} \quad (12)$$

Note that the construction of the right parts of the system (9) – (12) guarantees automatic fulfillment of conditions similar to those of (2), (4), (6), (8).

An analytical study of the system (9) – (12) is extremely difficult. For this reason, it was investigated numerically. In the next section, the results of its numerical study for a certain class of functions $\varphi_i(z)$ are presented.

3. Results of numerical experiments

Let us proceed to the presentation of the results of the numerical realization of the system (9) – (12), in which the functions $\varphi_i(z)$ were defined as follows:

$$\varphi_i(z_i) = a_i(1 - z_i), \quad i = 0, 1, \dots, m, m + 1. \quad (13)$$

The parameter $a_i > 0$, which participates in the definition of the function $\varphi_i(z)$, is a characteristic of the ability of the station with the number i to

increase cargo traffic. From the definition of the function $\varphi_i(z)$ it follows that the parameter d_0 , which is a characteristic of the demand for transportation, can be represented as follows:

$$d_0 = \mu a_0, \quad 0 < \mu \leq 1. \quad (14)$$

Let us rewrite the system of differential equations (9) – (12), in which the functions $\varphi_i(z)$ are defined according to (13), and the parameter d_0 is defined according to (14):

$$\begin{aligned} \dot{z}_0(t) = & \min(\mu a_0, a_0(1 - z_0(t))) \\ & - \min(z_0(t) + \min(\mu a_0, a_0(1 - z_0(t))), \\ & \lambda_1 a_1(1 - z_1(t))), \quad t \in [t_0, +\infty), \end{aligned} \quad (15)$$

$$\begin{aligned} \dot{z}_1(t) = & \min(z_0(t) + \min(\mu a_0, a_0(1 - z_0(t))), \\ & \lambda_1 a_1(1 - z_1(t))) - \min(z_1(t) + \min(z_0(t) \\ & + \min(\mu a_0, a_0(1 - z_0(t))), \lambda_1 a_1(1 - z_1(t))), \\ & \lambda_2 a_2(1 - z_2(t))), \quad t \in [t_0, +\infty), \end{aligned} \quad (16)$$

...

$$\begin{aligned} \dot{z}_i(t) = & \min(z_{i-1}(t) + \min(z_{i-2}(t) \\ & + \dots + \min(z_0(t) + \min(\mu a_0, a_0(1 - z_0(t))), \\ & \lambda_1 a_1(1 - z_1(t))), \lambda_2 a_2(1 - z_2(t))), \dots, \lambda_i a_i(1 - z_i(t))) \\ & - \min(z_i(t) + \min(z_{i-1}(t) + \dots + \min(z_0(t) \\ & + \min(\mu a_0, a_0(1 - z_0(t))), \lambda_1 a_1(1 - z_1(t))), \\ & \lambda_2 a_2(1 - z_2(t))), \dots, \lambda_{i+1} a_{i+1}(1 - z_{i+1}(t))), \\ & t \in [t_0, +\infty), \end{aligned} \quad (17)$$

...

$$\begin{aligned} \dot{z}_{m+1}(t) = & \min(z_m(t) + \min(z_{m-1}(t) \\ & + \dots + \min(z_0(t) + \min(\mu a_0, a_0(1 - z_0(t))), \\ & \lambda_1 a_1(1 - z_1(t))), \lambda_2 a_2(1 - z_2(t))), \dots, \\ & \lambda_{m+1} a_{m+1}(1 - z_{m+1}(t))) - \min(z_{m+1}(t) \\ & + \min(z_m(t) + \dots + \min(z_0(t) \\ & + \min(\mu a_0, a_0(1 - z_0(t))), \lambda_1 a_1(1 - z_1(t))), \\ & \lambda_2 a_2(1 - z_2(t))), \dots, a_{m+1}(1 - z_{m+1}(t))), \end{aligned}$$

$$d_{m+1}), \quad t \in [t_0, +\infty). \quad (18)$$

Here $\mu > 0$, $a_0 > 0$, $a_i > 0$, $0 < \lambda_i \leq 1$, $i = 1, \dots, m+1$, d_{m+1} are the parameters of the model:

μ is a characteristic of the range of demand for transportation, which can be satisfied with the existing technical potential of the stations;

a_i , $i = 0, 1, \dots, m+1$ is characteristic of the ability of a station with the number i to increase cargo traffic;

λ_i is a characteristic of the utilization degree of the technical potential of the station with the number i ;

d_{m+1} is a characteristic of the cargo distribution mode from the final node station.

Note that the parameters λ_i , $i = 1, \dots, m+1$ and d_{m+1} , unlike the others, are controllable and can be defined at any level within a given range, depending on the values of the remaining parameters.

The main purpose of the study of the system (15) – (18): for the given values of the inconsistency degrees of stations at the initial time ($0 \leq z_i(t_0) \leq 1$, $i = 0, 1, \dots, m+1$), characteristics of the demand for cargo transportation (parameter μ), characteristics of the ability of stations to increase cargo traffic (parameters a_i , $i = 0, 1, \dots, m+1$) to determine such values of the utilization degree of the technical potential of stations (parameters λ_i , $i = 1, 2, \dots, m+1$) and the cargo distribution mode from the final node station (parameter d_{m+1}) at which the inconsistency degree of all stations over time will be minimal.

The numerical realization of the system of differential equations (15) – (18) allowed us to investigate the properties of its solutions. The main result of this study is presented in the form of the following two statements.

Statement 1. *The system of differential equations (15)–(18) has an infinite set of stationary solutions of the form*

$$z_i(\cdot) \equiv c_i, \quad 0 \leq c_i \leq 1, \quad i = 0, 1, \dots, m+1,$$

and its arbitrary solution converges to one of them depending on the parameters μ , a_i , λ_i , $i = 0, 1, \dots, m+1$, d_{m+1} and initial values.

Let us introduce the notation

$$d = \lambda_k a_k = \min(\lambda_1 a_1, \lambda_2 a_2, \dots, \lambda_{m+1} a_{m+1})$$

and distinguish the following stationary solutions of the system (15) – (18):

$$z_i(\cdot) \equiv 0, \quad i = 0, 1, \dots, m+1; \quad (19)$$

$$z_0(\cdot) \equiv 1 - \frac{\lambda_k a_k}{a_0}, \quad z_i(\cdot) \equiv 1 - \frac{\lambda_k a_k}{\lambda_i a_i}, \quad i = 1, \dots, m+1; \quad (20)$$

$$z_0(\cdot) \equiv 1 - \frac{\lambda_k a_k}{a_0}, \quad z_i(\cdot) \equiv \begin{cases} 1 - \frac{\lambda_k a_k}{\lambda_i a_i}, & i = 1, \dots, k-1, \\ 0, & i = k, \dots, m+1; \end{cases} \quad (21)$$

$$z_0(\cdot) \equiv 1 - \frac{\lambda_k a_k}{a_0}, \quad z_i(\cdot) \equiv \begin{cases} 1 - \frac{\lambda_k a_k}{\lambda_i a_i}, & i = 1, \dots, k-1, \\ 0, & i = k, \dots, m; \end{cases},$$

$$0 \leq z_{m+1}(\cdot) \leq 1 - \frac{d_{m+1}}{\lambda_{m+1} a_{m+1}}; \quad (22)$$

$$z_0(\cdot) \equiv 1 - \frac{d_{m+1}}{a_0}, \quad z_i(\cdot) \equiv 1 - \frac{d_{m+1}}{\lambda_i a_i}, \quad i = 1, 2, \dots, m+1. \quad (23)$$

Each of these stationary solutions exists under certain conditions, which must satisfy the parameters of the system of equations (15) – (18). Here are these conditions:

$$d_0 < d \quad \text{and} \quad d_0 < d_{m+1} \quad (\text{solution (19)}), \quad (24)$$

$$d < d_0 \quad \text{and} \quad d < d_{m+1} \quad (\text{solutions (20) and (21)}), \quad (25)$$

$$d_{m+1} < d_0 \quad \text{and} \quad d_{m+1} < d \quad (\text{solution (21)}), \quad (26)$$

$$d_0 = d = d_{m+1} \quad (\text{solutions (19), (20), (21), (22), (23)}). \quad (27)$$

The following statement clarifies the nature of the solutions of the system of equations (15) – (18).

Statement 2. For any initial values

$$0 \leq z_i(t_0) \leq 1, \quad i = 0, 1, \dots, m+1,$$

parameters

$$0 < \mu \leq 1, \quad a_i > 0, \quad 0 < \lambda_i \leq 1, \quad i = 0, 1, \dots, m+1, \quad d_{m+1} > 0$$

the solution of the system (15) – (18) converges to:

- stationary solution (19) under condition (24);
- stationary solution (21) under condition (25);
- stationary solution (23) under condition (26).

Fig. 1, Fig. 2 and Fig. 3 show the convergence to the indicated stationary solutions.

Fig. 1. Convergence to a stationary solution (19)
 $(d_0 = 1.6, d = 1.8, d_{m+1} = 1.9)$

Fig. 2. Convergence to a stationary solution (21)
 $(d_0 = 1.9, d = 1.8, d_{m+1} = 1.95)$

Fig. 3. Convergence to a stationary solution (23)
 $(d_0 = 1.9, d = 1.8, d_{m+1} = 1.6)$

Thus, the stationary solutions (19), (21) and (23) under the corresponding restrictions on the parameters of the system of equations (15) – (18) (conditions (24), (25), (26)) are globally stable. The system of equations (15) – (18) has no other globally stable stationary solutions.

Using the result of Statement 2, we define such values of the parameters λ_i , $i = 1, 2, \dots, m + 1$ and d_{m+1} at which the inconsistency degree of all stations over time will be minimal.

Let us describe this procedure:

- a) Define $a_j = \min(a_1, a_2, \dots, a_{m+1})$;
- b) assign the maximum value from the specified range to the parameter λ_j , i.e. $\lambda_j = 1$;
- c) define the remaining parameters λ_i as follows: $\lambda_i = \frac{a_j}{a_i}$, $i = 1, \dots, m + 1$, $i \neq j$;
- d) match d_{m+1} in such a way that $d_{m+1} \geq \max(d_0, d)$, where $d_0 = \mu a_0$, $d = a_j$.

After performing this procedure, the parameters of the model λ_i , a_i , $i = 1, 2, \dots, m + 1$ and d_{m+1} will satisfy the following conditions:

$$\lambda_1 a_1 = \lambda_2 a_2 = \dots = \lambda_{m+1} a_{m+1} = d. \quad (28)$$

$$d_{m+1} \geq \max(d_0, d). \quad (29)$$

It follows from Statement 2 that if the conditions (28) – (29) are satisfied, an arbitrary solution of the system (15) – (18) tends to:

- a stationary solution (19) if $d_0 < d$;
- to a stationary solution

$$z_0(\cdot) \equiv 1 - \frac{d}{a_0}, \quad z_i(\cdot) \equiv 0, \quad i = 1, \dots, m+1, \quad \text{if } d_0 > d. \quad (30)$$

Further numerical investigation of the system (15) – (18) allowed us to obtain a more general result. We will give it in the following statement.

Statement 3. *For any initial values*

$$0 \leq z_i(t_0) \leq 1, \quad i = 0, 1, \dots, m+1,$$

parameters

$$0 < \mu \leq 1, \quad a_i > 0, \quad 0 < \lambda_i \leq 1, \quad d_{m+1} > 0,$$

satisfying the condition (28) – (29), the solution of the system (15) – (18) converges to:

- *stationary solution (19) if $d_0 < d$;*
- *stationary solution (30) otherwise.*

Thus, by controlling the parameters λ_i , $i = 1, 2, \dots, m+1$ and d_{m+1} (i.e., by correctly setting the utilization degree of the technical potential of the stations and the mode of cargo distribution from the final node station), it can be achieved that over time the inconsistency degree between reception and dispatch of goods at least at all stations except the initial one will be equal to 0.

4. Conclusion

This article presents a model of the organization of railway cargo transportation between two node stations. Such a model is given by a system of differential equations with a number of parameters characterizing the demand for cargo transportation, the ability of stations to increase cargo traffic and the degree of use of their technical potential, as well as the mode of cargo distribution from the final node station. Due to the complexity of the analytical study, its numerical realization was carried out. With the help of it, the properties of solutions of the specified system have been revealed, stationary solutions have been investigated on stability. The main result of this study is as follows: for the given values of the inconsistency degrees of stations at the initial moment of time, the demand for cargo transportation and the characteristics of the ability of stations to increase cargo traffic, such values of the utilization degrees of the technical potential of stations and the mode of cargo distribution from the

final node station are determined at which the inconsistency degree between the reception and dispatch of goods at all stations will eventually be minimal.

References

- [1] B. Harris, A.G. Wilson, Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models, *Environment and Planning A*, **10**, No 4 (1978), 371-388.
- [2] T. Leventhal, G.L. Nemhauser, L. Trotter, A column generation algorithm for optimal traffic assignment, *Transportation Science*, **7**, No 2 (1973), 168-176.
- [3] H.K. Lo, A. Chen, Traffic equilibrium problem with route-specific costs: formulation and algorithms, *Transportation Research: Part B*, **34**, No 6 (2000), 493-513.
- [4] H. Bar-Gera, Origin-based algorithm for the traffic assignment problem, *Transportation Science*, **36**, No 4 (2002), 398-417.
- [5] C.F. Daganzo, The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory, *Transportation Research B*, **28**, No 4 (1994), 269-287.
- [6] M. Bando, K. Hasebe, A. Nakayama, A. Shibata, Y. Sugiyama, Dynamical model of traffic congestion and numerical simulation, *Physical Review E*, **51**, No 2 (1995), 1035-1042.
- [7] M. Treiber, A. Hennecke, D. Helbing, Congested traffic states in empirical observations and microscopic simulations, *Physical Review E*, **62**, No 2 (2000), 1805-1824.
- [8] D.Yu. Levin, Modelirovanie protsessov perevozki [Modeling of transportation processes], *World of Transport and Transportation*, **8**, No 5 (2010), 48-55 (in Russian).
- [9] G. de Jong, H.F. Gunn, W. Walker, National and international freight transport models: an overview and ideas for further development, *Transport Reviews*, **24**, No 1 (2004), 103-124.
- [10] T. Yamada, B.F. Russ, J. Castro, E. Taniguchi, Designing multimodal freight transport networks: A heuristic approach and applications, *Transportation Science*, **43**, No 2 (2009), 129-143.

- [11] L.A. Beklaryan, N.K. Khachatryan, Traveling wave type solutions in dynamic transport models, *Functional Differential Equations*, **13**, No 12 (2006), 125-155.
- [12] L.A. Beklaryan, N.K. Khachatryan, On one class of dynamic transportation models, *Computational Mathematics and Mathematical Physics*, **53**, No 10 (2013), 1466-1482.
- [13] N.K. Khachatryan, A.S. Akopov, Model for organizing cargo transportation with an initial station of departure and a final station of cargo distribution, *Business Informatics*, **39** (2017), 25-35.
- [14] N.K. Khachatryan, A.S. Akopov, F.A. Belousov, About quasi-solutions of traveling wave type in models for organizing cargo transportation, *Business Informatics*, **43** (2018), 61-70.
- [15] L.A. Beklaryan, N.K. Khachatryan, A.S. Akopov, Model for organization cargo transportation at resource restrictions, *International Journal of Applied Mathematics*, **32**, No 4 (2019), 627-640; DOI: 10.12732/ijam.v32i4.7.
- [16] N.K. Khachatryan, Study of flow dynamics in the model Model of cargo transportation organization beetwen node stations, *International Journal of Applied Mathematics*, **33**, No 5 (2020), 937-949; DOI: 10.12732/ijam.v33i5.14.