

SOLITON SOLUTIONS OF THE LOADED  
MODIFIED CALOGERO-DEGASPERIS EQUATION

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**Abstract:** In this article, we construct exact travelling wave solutions of the loaded modified Calogero-Degasperis equation by  $(G'/G)$  - expansion method. The efficiency of this method for finding these exact solutions has been demonstrated. We establish several classes of explicit solutions - hyperbolic and trigonometric solutions containing free parameters. The solitary wave solutions of this equation follow from the traveling wave solutions for certain values of the parameters. All calculations have been made with the aid of Matlab program. Our results reveal that the method is a very effective and straightforward way of formulating the exact travelling wave solutions of nonlinear wave equations arising in mathematical physics and engineering.

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**Key Words:** Calogero-Degasperis equation; soliton solution; hyperbolic function; the loaded modified equation; an expansion method

## 1. Introduction

Nonlinear evolution equations (NLEE) appear in various fields of science and technology, such as fluid mechanics, plasma physics, optical fibers, biophysics, electricity, wave propagation in shallow water, high energy physics, biology, solid state physics, etc. One of the most important NPDE is the Calogero-

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Degasperis equation (CD)

$$u_{xt} + \alpha u_x u_{xy} + \beta u_y u_{xx} = 0, \quad (1)$$

where  $u(x, y, t)$  is an unknown function,  $x \in R$ ,  $y \in R$ ,  $t \geq 0$ ,  $\alpha$  and  $\beta$  are any constants.

This equation describes the (2+1) - dimensional interaction of the Riemann wave propagating along the  $y$  - axis with a long wave along the  $x$  - axis. The equation (1) was first constructed by Bogoyavlenskii and Schiff in different ways [1, 2, 3]. Bogoyavlenskii used the modified Lax formalism, where as Schiff derived the same equation by reducing the self-dual Yang-Mills equation. Many exact solutions have been found for equation (1) by using symmetry method [4]. In [5], the equation (1) has been solved by using Hirota's bilinear method.

In this paper, we consider the following the loaded modified Calogero-Degasperis equation

$$u_{xt} + \alpha u_x u_{xy} + \beta u_y u_{xx} + u_{xxx} + \gamma(t)u(0, 0, t)u_{xx} = 0, \quad (2)$$

where  $u(x, y, t)$  is an unknown function,  $x \in R$ ,  $y \in R$ ,  $t \geq 0$ ,  $\gamma(t)$  - is the given real continuous function.

One of the main physical problems for this model is to obtain their soliton solutions. We construct exact travelling wave solutions of the loaded modified Calogero-Degasperis equation by  $(G'/G)$  - expansion method. It is shown that the search for soliton solutions using the  $(G'/G)$  - extension method is one of the most effective methods for finding solutions to integrable nonlinear evolution equations due to the convenience of using well-known software packages compared to other well-known methods such as the Hirota's bilinear operations [6], inverse scattering transform [7], extended tanh-function method [8] and homogenous balance method [9].

In recent years, in connection with intensive research of problems optimal management of the agroecosystem, for example, the problem of long-term forecasting and regulation of the level of groundwater and soil moisture, there has been a significant increase in interest in loaded equations. Among the works devoted to loaded equations, one should especially note the works of A. Kneser [10], L. Lichtenstein [11], A. M. Nakhushev [12, 13], and others. It is known that the loaded differential equations contain some of the traces of an unknown function. In [14, 15, 16, 17], the term of "loaded equation" was used for the first time, the most general definitions of the loaded differential equation were given and also a detailed classifications of the differential loaded equations as well as their numerous applications were presented. A complete description of

solutions of the nonlinear loaded equations and their applications can be found in papers [18, 19, 20, 21, 22, 23].

## 2. Description of the generalized ( $G'/G$ )-expansion method

Consider nonlinear evolution equations with independent variables  $x$ ,  $y$  and  $t$  is of the form

$$F(u, u_x, u_y, u_t, u_{xx}, u_{tt}, u_{yy}, u_{xy}, u_{xt}, u_{yt} \dots) = 0, \quad (3)$$

$u = u(x, y, t)$  is a unknown function,  $F$  is a polynomial in  $u = u(x, y, t)$  and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. Now we give the main steps of the ( $G'/G$ ) - expansion method.

**Step 1.** By using traveling wave transformation

$$u(x, y, t) = u(\xi), \quad \xi = px + qy - \varphi(t), \quad (4)$$

where  $\dot{\varphi}(t)$  is the speed of the traveling wave. Now, using (4), the equation (2) is converted into an ordinary differential equation for  $u = u(\xi)$ :

$$P(u, u', u'', u''', \dots) = 0, \quad (5)$$

where  $P$  is a polynomial of  $u(\xi)$ .

**Step 2.** Suppose the solution of (5) can be expressed by a polynomial in ( $G'/G$ ) as follows:

$$u(\xi) = \sum_{j=0}^m a_j \left( \frac{G'}{G} \right)^j, \quad (6)$$

where  $G = G(\xi)$  satisfies the following second order linear ordinary differential equation

$$G'' + \lambda G' + \mu G = 0, \quad (7)$$

while  $a_j$ ,  $\lambda$  and  $\mu$  are constants.

**Step 3.** The positive integer  $m$  can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in (5). The coefficients of this polynomial can be obtained by solving a set of algebraic equations resulted from the process of using the method.

**Step 4.** Substitute (6) along with (7) into (5) and collect all terms with the same order of  $G'(\xi)/G(\xi)$ , the left-hand side of (5) is converted into a polynomial in  $G'(\xi)/G(\xi)$ . Next, equating the coefficients of each power of  $(G'/G)$  to zero, obtain a system of algebraic equations for  $a_j$  and  $\varphi(t)$ . Then,  $u(\xi)$  can be restored by (6).

### 3. Exact solutions of the loaded modified Calogero-Degasperis equation

We will show how to find the exact solution of the loaded modified Calogero-Degasperis equation using the  $(G'/G)$  - expansion method. For doing this, we perform the steps above for equation (2). The traveling wave transformation

$$u(x, y, t) = u(\xi), \quad \xi = px + qy - \varphi(t), \tag{8}$$

permits us converting equation (2) into an ordinary differential equation for  $u = u(\xi)$ :

$$-\dot{\varphi}u'' + \alpha p^2 qu'u'' + \beta p^2 qu'u'' + p^3 qu^{IV} + p\gamma(t)u(0, 0, t)u'' = 0. \tag{9}$$

Integrating (9) with respect to  $\xi$ , we get

$$-\dot{\varphi}u' + \frac{p^2q(\alpha + \beta)}{2}u'^2 + p^3qu''' + \eta(t)u' = 0, \tag{10}$$

where  $\eta(t) = p\gamma(t)u(0, 0, t)$ .

We search the solution of equation (10) in the form of a polynomial in  $(G'/G)$  as

$$u(\xi) = \sum_{j=0}^m a_j \left(\frac{G'}{G}\right)^j, \tag{11}$$

where  $G = G(\xi)$  satisfies the second order ordinary differential equation in the form

$$G'' + \lambda G' + \mu G = 0. \tag{12}$$

Using (11) and (12),  $u'^2$  and  $u'''$  are easily derived the following equalities

$$u'^2(\xi) = m^2 a_m^2 \left(\frac{G'}{G}\right)^{2m+2} + \dots, \tag{13}$$

$$u'''(\xi) = m(m + 1)(m + 2)a_m \left(\frac{G'}{G}\right)^{m+3} + \dots \tag{14}$$

Considering the homogeneous balance between  $u'''$  and  $u'^2$  in equation (10), based on (13) and (14) we required that  $m + 3 = 2m + 2 \Rightarrow m = 1$ . It follows that,  $u(\xi)$  has the form

$$u(\xi) = a_0 + a_1 \left( \frac{G'}{G} \right). \tag{15}$$

By introducing the notation

$$Y = Y(\xi) = \frac{G'}{G}$$

and the help of (12), we get following equalities

$$Y' = \frac{GG'' - G'^2}{G^2} = \frac{G(-\lambda G' - \mu G) - G'^2}{G^2} = -Y^2 - \lambda Y - \mu,$$

$$Y'' = 2Y^3 + 3\lambda Y^2 + (2\mu + \lambda^2)Y + \lambda\mu,$$

$$Y''' = -6Y^4 - 12\lambda Y^3 - (7\lambda^2 + 8\mu)Y^2 - (\lambda^3 + 8\lambda\mu)Y - (\lambda^2\mu + 2\mu^2).$$

Then, collecting all terms of same order of  $Y$  in equation (10) and equating to zero, yields a set of algebraic equations for  $a_0, a_1, a_2, \dots, a_m$ .

It is known that the solution of equation (12) is a linear combination of sinh and cosh. For instance, in case  $(\lambda^2 - 4\mu) > 0$ , the solution of equation (12) has the form

$$G(\xi) = e^{-\frac{\lambda\xi}{2}} \left( A \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + B \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi \right),$$

$$\frac{G'(\xi)}{G(\xi)} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \frac{A \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + B \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{A \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + B \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi},$$

where A and B are any constants.

It is known that the exact view of  $u'$ ,  $u'^2$  and  $u'''$  are easily defined as follows

$$u'(\xi) = -a_1 Y^2 - a_1 \lambda Y - a_1 \mu, \tag{16}$$

$$u'^2(\xi) = a_1^2 Y^4 + a_1^2 \lambda^2 Y^2 + a_1^2 \mu^2 + 2a_1^2 \lambda Y^3 + 2a_1^2 \mu Y^2 + 2a_1^2 \lambda \mu Y, \tag{17}$$

$$u'''(\xi) = -6a_1 Y^4 - 12a_1 \lambda Y^3 - (8a_1 \mu + 7a_1 \lambda^2) Y^2 - (8a_1 \lambda \mu + a_1 \lambda^3) Y - (2a_1 \mu^2 + a_1 \lambda^2 \mu). \tag{18}$$

By substituting (15)-(18) into equation (10) and collecting all terms with the same power of  $Y$ , the left-hand side of equation (10) is converted into

another polynomial in  $Y$  and equating each coefficient of expression (10) to zero, yields a set of simultaneous equations for  $a_0$ ,  $a_1$ ,  $\varphi(t)$  and  $C$  as following:

$$\begin{aligned} \frac{p^2q(\alpha + \beta)}{2}a_1^2 - 6a_1p^3q &= 0, \\ a_1^2\lambda p^2q(\alpha + \beta) - 12a_1\lambda p^3q &= 0, \\ \frac{a_1^2\lambda^2 p^2q(\alpha + \beta)}{2} + a_1^2\mu p^2q(\alpha + \beta) - 8a_1p^3q\mu - \\ - 7a_1p^3q\lambda^2 + a_1(\dot{\varphi} - \eta(t)) &= 0, \\ a_1^2\lambda\mu p^2q(\alpha + \beta) - 8a_1p^3q\lambda\mu - a_1p^3q\lambda^3 + a_1\lambda\dot{\varphi} - a_1\lambda\eta(t) &= 0, \\ \frac{p^2q(\alpha + \beta)}{2}a_1^2\mu^2 - 2a_1p^3q\mu^2 - a_1p^3q\lambda^2\mu + a_1\mu\dot{\varphi} - a_1\mu\eta(t) &= 0. \end{aligned}$$

By solving these equations, we obtain

$$\begin{aligned} a_0 = C, a_1 &= \frac{12p}{\alpha + \beta}, \\ \varphi &= p^3q(\lambda^2 - 4\mu)t + \eta(t), \end{aligned}$$

$$\varphi(t) = p^3q(\lambda^2 - 4\mu)t + \int_0^t \eta(\tau)d\tau + \varphi^0, \quad (19)$$

where  $\lambda$ ,  $\mu$ ,  $p$ ,  $q$ ,  $C$  and  $\varphi^0$  are arbitrary constants. Using (19), expression (15) can be rewritten as

$$u(\xi) = C + \frac{12p}{\alpha + \beta}Y, \quad (20)$$

$$\frac{G'(\xi)}{G(\xi)} = -\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2} \frac{A \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + B \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi}{A \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi + B \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2}\xi},$$

where  $\xi = px + qy - p^3q(\lambda^2 - 4\mu)t - \int_0^t \eta(\tau)d\tau - \varphi^0$ .

The function (20) is a solution of equation. Substituting the general solutions of equation (12) into (20), we have several types of travelling wave solutions of the loaded modified Calogero-Degasperis equation (2) as follows:

1) When  $(\lambda^2 - 4\mu) > 0$ , we have

$$u(\xi) = C - \frac{6\lambda p}{\alpha + \beta} + \frac{6p\sqrt{\lambda^2 - 4\mu}}{\alpha + \beta} \times$$

$$\times \frac{A \sin h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + B \cos h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}{A \cosh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi + B \sinh \frac{\sqrt{\lambda^2 - 4\mu}}{2} \xi}. \tag{21}$$

It is obvious that the function  $u(0, 0, t)$  can be easily found based on expression (21). For example, let  $\gamma(t)$  be given to get

$$\begin{aligned} \xi &= \sum_{j=1}^n \alpha_j t^j, \quad \xi' = \sum_{j=1}^n j \alpha_j t^{j-1}, \\ \xi' &= -p^3 q (\lambda^2 - 4\mu) - p \gamma(t) u(0, 0, t), \\ \gamma(t) &= \frac{-\frac{1}{p} \sum_{j=1}^n j \alpha_j t^{j-1} - p^2 q (\lambda^2 - 4\mu)}{C - \frac{6\lambda p}{\alpha + \beta} + \frac{6p \sqrt{\lambda^2 - 4\mu}}{\alpha + \beta} \frac{\sum_{j=1}^n (A \sin h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j + B \cos h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j)}{\sum_{j=1}^n (A \cos h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j + B \sin h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j)}}, \end{aligned}$$

where  $\alpha_j$  ( $j = 1, 2, \dots, n$ ) are constants. If  $A \neq 0, B = 0$  and  $(\lambda^2 - 4\mu) > 0$ , then  $u(x, y, t)$  becomes

$$\begin{aligned} u(x, y, t) &= C - \frac{6\lambda p}{\alpha + \beta} + \frac{6p \sqrt{\lambda^2 - 4\mu}}{\alpha + \beta} \\ &\times th \frac{\sqrt{\lambda^2 - 4\mu}}{2} \left(x - \sum_{j=1}^n \alpha_j t^j\right). \end{aligned} \tag{22}$$

The function (22) is the solution of the following the loaded modified Calogero-Degasperis equation

$$\begin{aligned} &u_t + u^2 u_x + u_{xxx} + u_{xyy} \\ &+ \frac{-\frac{1}{p} \sum_{j=1}^n j \alpha_j t^{j-1} - p^2 q (\lambda^2 - 4\mu) u(0, 0, t) u_x}{C - \frac{6\lambda p}{\alpha + \beta} + \frac{6p \sqrt{\lambda^2 - 4\mu}}{\alpha + \beta} \frac{\sum_{j=1}^n (A \sin h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j + B \cos h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j)}{\sum_{j=1}^n (A \cos h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j + B \sin h \frac{\sqrt{\lambda^2 - 4\mu}}{2} \alpha_j t^j)}} = 0. \end{aligned}$$

2) When  $(\lambda^2 - 4\mu) < 0$ , we have

$$\begin{aligned} u(\xi) &= C - \frac{6\lambda p}{\alpha + \beta} - \frac{6p \sqrt{4\mu - \lambda^2}}{\alpha + \beta} \\ &\times \frac{A \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + B \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}{A \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \xi + B \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \xi}. \end{aligned} \tag{23}$$

It is not difficult to find  $u(0, 0, t)$  based on expression (23). Let  $\gamma(t)$  be given to have

$$\begin{aligned} \xi &= \sum_{j=1}^n \alpha_j t^j, \quad \xi' = \sum_{j=1}^n j \alpha_j t^{j-1}, \\ \xi' &= -p^3 q(\lambda^2 - 4\mu) - p\gamma(t)u(0, 0, t), \\ \gamma(t) &= \frac{-\frac{1}{p} \sum_{j=1}^n j \alpha_j t^{j-1} - p^2 q(\lambda^2 - 4\mu)}{C - \frac{6\lambda p}{\alpha + \beta} - \frac{6p\sqrt{4\mu - \lambda^2}}{\alpha + \beta} \frac{\sum_{j=1}^n (A \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j + B \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j)}{\sum_{j=1}^n (A \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j + B \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j)}}, \end{aligned}$$

where  $\alpha_j$  ( $j = 1, 2, \dots, n$ ) are constants. If  $A \neq 0$  and  $B = 0$ , then  $u(x, y, t)$  becomes

$$\begin{aligned} u(x, y, t) &= C - \frac{6\lambda p}{\alpha + \beta} - \frac{6p\sqrt{4\mu - \lambda^2}}{\alpha + \beta} \sqrt{4\mu - \lambda^2} \\ &\times tg \frac{\sqrt{4\mu - \lambda^2}}{2} (x - \sum_{j=1}^n \alpha_j t^j). \end{aligned} \tag{24}$$

The function (24) is the solution of the following the loaded modified Calogero-Degasperis equation

$$\begin{aligned} &u_t + u^2 u_x + u_{xxx} + u_{xyy} \\ &+ \frac{-\frac{1}{p} \sum_{j=1}^n j \alpha_j t^{j-1} - p^2 q(\lambda^2 - 4\mu)u(0, 0, t)u_x}{C - \frac{6\lambda p}{\alpha + \beta} + \frac{6p\sqrt{4\mu - \lambda^2}}{\alpha + \beta} \frac{\sum_{j=1}^n (A \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j + B \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j)}{\sum_{j=1}^n (A \cos \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j + B \sin \frac{\sqrt{4\mu - \lambda^2}}{2} \alpha_j t^j)}} = 0. \end{aligned}$$

3) When  $(\lambda^2 - 4\mu) = 0$ , we have

$$u(\xi) = C - \frac{6\lambda p}{\alpha + \beta}. \tag{25}$$

#### 4. Graphical representation of the loaded modified Calogero-Degasperis equation

Graphical representation is an effective tool for communication and it exemplifies evidently the solutions of the problems. The graphical illustrations of the solutions are depicted in Figure 1 and Figure 2. After visualizing the graphs of the soliton solutions and the periodic wave solutions by using distinct values of random parameter are demonstrated to better understand their physical



features. The amplitude and velocities are controlled by parameters of various kind. The soliton is a self-reinforcing wave packet maintaining its shape while propagating at a constant velocity. Solitons are unscathed in shape and speed by a collision with other solitons and are often studied in quantum mechanics, nuclear physics, and waves along a weakly anharmonic mass-spring chain.

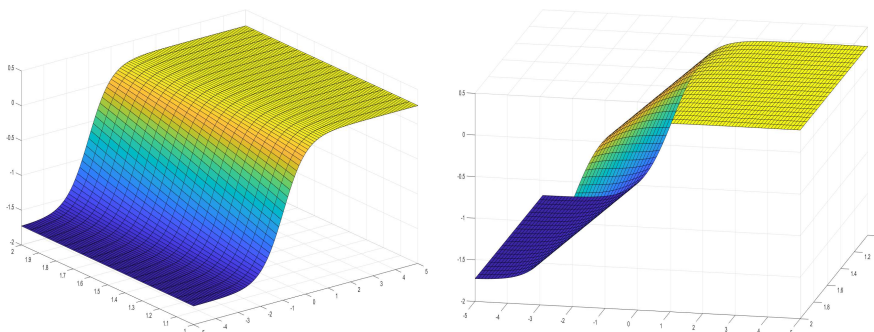


Figure 1: The soliton solution of the loaded modified Calogero-Degasperis equation for  $\lambda = 2\sqrt{3}$ ,  $\mu = 2$ ,  $C = 1$ ,  $p = 1$ ,  $\alpha = 4$ ,  $\beta = 8$ .

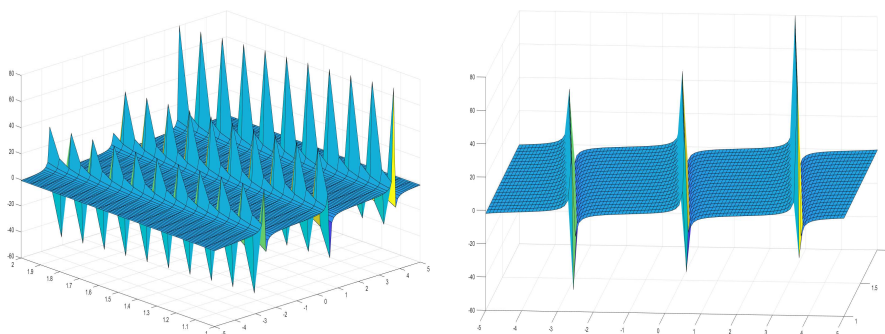


Figure 2: The periodic solution of the loaded modified Calogero-Degasperis equation for  $\lambda = 2$ ,  $\mu = 2$ ,  $C = 1$ ,  $p = 1$ ,  $\alpha = 4$ ,  $\beta = 8$ .

## 5. Conclusions

In this article, we presented the travelling wave solutions in terms of hyperbolic and trigonometric functions for the loaded modified Calogero-Degasperis equation. The exactness of the obtained results is studied by using software MATLAB program. The received solutions with free parameters may be important to explain some physical phenomena. It is shown that the performance of this method is productive, effective and well-built mathematical tool for solving nonlinear evolution equations.

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